Diffractive optical elements based on Fourier optical techniques: a new class of optics for extreme ultraviolet and soft x-ray wavelengths

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A diffractive optical element, based on Fourier optics techniques, for use in extreme ultraviolet/soft x-ray experiments has been fabricated and demonstrated. This diffractive optical element, when illuminated by a uniform plane wave, will produce two symmetric off-axis first-order foci suitable for interferometric experiments. The efficiency of this optical element and its use in direct interferometric determination of optical constants are also discussed. Its use in direct interferometric determination of optical constants is also referenced. Its use opens a new era in the use of sophisticated optical techniques at short wavelengths. © 2002 Optical Society of America

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1. Introduction

Coherent extreme ultraviolet (EUV) and soft x-ray radiation facilitates phase-sensitive techniques that provide new opportunities in various fields, e.g., biological imaging, material characterization, and nanotechnology. However, challenges are presented in that very limited optical elements are available at these wavelengths. No appropriate materials exist for lenses and prisms due to high absorption. Most experiments either utilize low efficiency diffractive optics such as Fresnel zoneplates, or glancing incidence reflection mirrors and normal incidence multilayer mirrors that result in restrictive off-axis optical systems and a limited spectral region, respectively. Therefore, devising novel optical elements that can effectively and efficiently achieve wave-front shaping is of crucial importance for researches conducted at EUV/SXR wavelengths. Here, Fourier optical techniques are introduced to accomplish the desired wave-front manipulation.

In our first example of these techniques, which are new to the best of our knowledge, we have designed and fabricated, based on Fourier optics techniques, a diffractive optical element that combines the functions of the grating and zone plate through a bit-wise eXclusive OR (XOR) operation. By use of this compound diffractive optical element allows the efficiency and the contrast of the interferometer to be greatly increased. This optical element has been used in an EUV interferometer to directly determine the index of refraction at EUV wavelengths. Similar activities are underway at soft x-ray wavelengths.

2. XOR Pattern

This XOR diffractive optical element is obtained by combining a 50% duty-cycle binary intensity grating and a 50% duty-cycle intensity zone plate. The binary grating and zone plate are first pixelized, with each pixel being either 1 or 0 for transmission and absorption, respectively. The two pixelized patterns are then overlapped and compared pixel by pixel to produce the resulting XOR pattern, i.e., at each pixel position, if the pixel values of the grating and zone plate are the same (both 0’s or both 1’s), the value of the corresponding pixel on the XOR pattern is 0. Otherwise, the value of the corresponding pixel on the XOR pattern is 1.

For a 50% duty-cycle grating of period d, the transmitted intensity function is

\[ G(x, y) = \frac{1}{2} [1 + \text{sgn}(\cos \nu x)], \] (1)
where $v = 2\pi/d$.

Similarly, for a 50% duty-cycle zoneplate of diameter $D$ and outermost zone width $\Delta r$, the transmitted intensity function is

$$ZP(x, y) = \frac{1}{2} [1 + \text{sgn}(\cos \gamma r^2)], \quad (2)$$

where $r = \sqrt{x^2 + y^2}$ and $\gamma = [\pi/\Delta r(D - \Delta r)]$.

Expand these two patterns in their Fourier series,

$$G(x, y) = \sum_{m=-\infty}^{\infty} \frac{\sin(m\pi/2)}{m\pi} \exp(-jmvx), \quad (3)$$

$$ZP(x, y) = \sum_{n=-\infty}^{\infty} \frac{\sin(n\pi/2)}{n\pi} \exp(-j\gamma r^2). \quad (4)$$

Note that by comparing the Fourier series of a zoneplate with a lens, one finds that the zoneplate functions as multiple lenses with $n$th order focal length $f_n = (-\pi/n\lambda\deltay)$.

The XOR pattern of the combined grating and the zoneplate is obtained by forming

$$\text{XOR}(x, y) = G(x, y) + ZP(x, y) - 2G(x, y)ZP(x, y)$$

$$= \sum_{m=-\infty}^{\infty} \frac{\sin(m\pi/2)}{m\pi} \exp(-jmvx) + \sum_{n=-\infty}^{\infty} \frac{\sin(n\pi/2)}{n\pi} \exp(-j\gamma r^2) - 2\left[\frac{1}{2} + \sum_{m=-\infty}^{\infty} \frac{\sin(m\pi/2)}{m\pi} \exp(-jmvx)\right] \times \left[\frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{\sin(n\pi/2)}{n\pi} \exp(-j\gamma r^2)\right]. \quad (5)$$

This combined diffractive element, when illuminated by a uniform wavefront, has the interesting property that it produces two symmetric off-axis focal spots, $(m, n) = (\pm 1, 1)$, at the back focal plane of the zoneplate. Note that both the grating and the zoneplate have to be of 50% duty cycle for the on-axis focal spot to disappear, i.e., $m \neq 0$ and $n \neq 0$ in the summation. The separation of these two beam spots $\Delta x$ can be determined by multiplying the two exponentials in Eq. (5), completing the square for $x$-terms, thus resulting in $\Delta x = 2\Delta r(D - \Delta r)/d \approx 2\Delta r D/d$. Note that this separation is independent of wavelength $\lambda$. Thus as the wavelength is varied for spectral determination of the index of refraction, the focal length (distance from the XOR pattern to the sample mask) varies, but the lateral separation of the two beam spots remains fixed. The invariance of the spot separation over the wavelength allows the EUV interferometer to operate at different wavelengths without the need of changing the image-plane sample mask. This is a desirable property for EUV interferometers because the scale of the sample mask for EUV applications requires it to be micro or nano fabricated, and thus immutable after being made.

**Simulation of the XOR Pattern.** A computer simulation is performed to see if these patterns produce the expected results. An XOR pattern of a grating (period $d = 16 \text{ m}$) and a zoneplate (outermost zone-width $\Delta r = 0.2 \text{ m}$, diameter $D = 400 \text{ m}$) is produced, as shown in Fig. 1(a). This pattern is then Fresnel-propagated to the first order focal plane of the zoneplate and the resulting intensity distribution is shown in Fig. 1(b). As expected, only two off-axis spots exist in this focal plane.

### 3. Efficiency of the XOR Pattern

The XOR pattern, as expressed in Eq. (5) gives the efficiencies of the individual orders. First, we need to determine the overall transparent area of this XOR pattern. Because we know that the percent of the transparent area on the grating and the zoneplate is 1/2, we find that the overall transparent area of the XOR pattern to be $1/2 + 1/2 - 2(1/2)(1/2) = 1/2$ from Eq. (5).

Next, we calculate the efficiency of individual orders from their relative strength. From Eq. (5), we have, for $m, n \neq 0$,

$$\eta_{m,n} = \begin{cases} \frac{4}{m^2 n^2 \pi^4} & \text{if } m, n \text{ are both even}, \\ 0 & \text{if } m \text{ or } n \text{ is even}, \end{cases} \quad (6)$$

where $\sum_{k=0}^{\infty} 1/(2k + 1)^2 = (\pi^2/8)$ is used in the calculation.

Another way to look at this is that we can think of this XOR pattern as a binary amplitude zoneplate, multiplied by a $\pi$-phase-shift grating that does not have any absorption. Therefore the overall absorption of this XOR pattern is the same as that of a binary amplitude zoneplate, i.e., 1/2 and the efficiency of its individual orders is given by multiplying the corresponding orders of the binary amplitude zoneplate and the $\pi$-phase-shift grating. The efficiency $\eta_m$ of a 50% duty-cycle $\pi$-phase-shift grating is

$$\eta_m = \begin{cases} \frac{4}{m^2 \pi^2} & \text{for } m = \pm 1, \pm 3, \ldots, \\ 0 & \text{for } m \text{ is even}. \end{cases} \quad (7)$$

The efficiency $\eta_n$ of a binary amplitude zoneplate is

$$\eta_n = \begin{cases} \frac{1}{n^2 \pi^2} & \text{for } n = \pm 1, \pm 3, \ldots, \\ 0 & \text{for } n \text{ is even}. \end{cases} \quad (8)$$

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By comparing Eq. (6) with Eq. (7) and Eq. (8), we indeed see that the efficiency of the individual orders of the XOR pattern, $\eta_{m,n}$, is given by $\eta_m \times \eta_n$, i.e., the multiplication of the corresponding orders of the phase grating and amplitude zoneplate.

4. Visible Light Experiment

A first XOR pattern, designed for proof-of-principle testing at visible wavelengths, is fabricated using e-beam lithography to directly observe the intensity distribution at the back focal plane. The pattern is
defined by electroplating a 100-nm thickness of nickel, which is highly absorptive in both EUV and visible wavelengths. The grating used in this visible version has a period of 5 μm, the zoneplate diameter is 5 mm, and the outermost zone width is 2 μm. A screen is put at its back plane, which is 15.8 mm away from this visible XOR pattern. A collimated He–Ne laser beam (λ = 633 nm) is then used to illuminate this visible version of the XOR pattern and the resulting intensity distribution at the back focal plane is shown in Fig. 2(a). As expected, the two symmetric off-axis foci are directly observable and there is no on-axis focus presented. The separation between these two off-axis spots are measured to be 4 mm, which agrees with the designed value. As a comparison, an OR pattern made from the same grating and zoneplate is also fabricated and tested, as shown in Fig. 3. Combining the grating and zoneplate through a bit-wise OR operation is equivalent to placing them in tandem. Therefore this OR pattern demonstrates the back focal plane intensity distribution of a traditional separate grating and zoneplate setup. Figure 2(b) shows the resulting intensity distribution at the back focal plane of this OR pattern. Three foci are clearly observed, with the strongest focus on-axis and two weaker symmetric off-axis foci. The separation between the on-axis and the off-axis spots are measured to be 2 mm, which again agrees with the designed value.

5. First use in Extreme Ultraviolet Interferometry

The XOR pattern employed in our first application to EUV interferometry is fabricated using the same e-beam lithography tool, and a scanning electron microscopy image of the actual pattern is shown in Fig. 4. The grating has a period d of 16 μm and covers a 400 μm × 400 μm square area. The zoneplate has a diameter D = 400 μm and a outermost zone width Δr = 0.2 μm. Undulator beam line 12 at the advanced light source provides the EUV radiation for this measurement. The wavelength at which this measurement was performed is λ = 16.53 nm (75 eV) and the monochromator at the beam line is set at λ/Δλ = 1100.

This interferometer utilizes the strongest non-zeroth order, i.e., (m, n) = (±1, 1), which has a theoretical efficiency of 4/π² × 1/π² = 4/π⁴ ~ 4.1% as given by Eq. (6). Experimentally, the efficiency of this XOR pattern is measured by recording the total counts on the CCD while scanning a knife-like beam stop transversely across the back focal plane. Starting with the beam stop placed at the back focal plane such that the entire beam is blocked, as the beam stop slowly moves aside, allowing a fraction of light to pass, the total count on the CCD increases. The result of this efficiency measurement is shown in Fig. 5. The two abrupt steps at the center is caused by the two symmetric off-axis first order foci, (m, n) = (±1, 1), being released one at a time by the scanning
beam stop. However, when determining the efficiency of the \((m, n) = (\pm 1, 1)\) order, the effect of undiffracted straight-through light needs to be removed. Because the position of the transversely scanning beam stop is directly proportional to the fraction of the straight through light that passes it, the effect of straight through light can be determined by the constant slope of the two straight sections. After removing the effect of the straight through light by least-square fitting the slope of the two straight sections, the individual strength of the \((m, n) = (\pm 1, 1)\) order is shown to be around 4.0%, which agrees

![Graph](image)

**Fig. 5.** Efficiency of this XOR pattern is measured by scanning a knife-like beam stop across the focal plane. Starting with the beam stop placed at the back focal plane such that the entire beam is blocked, as the beam stop slowly moves aside, the total counts on the CCD increases, allowing fractions of light to pass. The constant slope of the two straight sections results from the effect of zeroth order (straight through) light. The two abrupt steps at the center is caused by the two symmetric off-axis first-order foci being released one at a time by the beam stop. Their strength is shown to be around 4.0%, which agrees with the theoretical value.

![Graph](image)

**Fig. 6.** Object wave, which consists of two converging spherical wavefronts, interferes with a reference plane wave, and the resulting intensity interference pattern is usually referred to as a CGH. This CGH is then binarized for nanofabrication by e-beam lithography. (a) Shows its binarized form. When illuminated by a uniform plane wave, this optical element reconstructs the object wave (two converging spherical waves) as shown in (b). Note that the two spots are symmetrically off-axis.
with the theoretical value. Note that the definition of diffraction efficiency for this element is the sum of the flux in the two desired orders divided by the total incident flux on the pattern. We measured the diffracted flux to the two desired orders and the total flux through the XOR pattern. The latter is assumed to be half of the total flux incident on the XOR pattern, as half the pattern is transparent. Therefore the diffraction efficiency is obtained by dividing the diffracted flux in the two orders by twice the total flux through the XOR pattern.

In comparison with the traditional separate binary grating and zoneplate setup, in which the ±1st orders of the grating are being focused by the first order of the zone plate with an overall efficiency of $1/\pi^2 \sim 1.0\%$, this XOR pattern provides a 4-times improvement in theory. In practice, the required exposure time is actually reduced by approximately 10 times because of the fact that the substrates on which these optical elements are fabricated have finite absorption, and only one substrate is needed in this case. This improvement in efficiency enables the first direct measurement of the refractive index at EUV wavelengths, where the two symmetric first-order foci are used as arms of an interferometer, and therefore enables a direct phase measurement for the dispersive part of the index of refraction.

6. Comparison with the Computer Generated Hologram

A computer generated hologram (CGH) having similar functions can be constructed by encoding the object wave, which consists of two converging spherical wavefronts by a reference plane wave to form an interference pattern (hologram). This CGH, when illuminated by a reference plane wave, would produce two converging spherical wavefronts that can be used for interferometric experiments. These two spherical wavefronts would be identical and symmetrically distributed with respect to the optical axis.

To nanofabricate this CGH, it is necessary to binarize the smooth area interference pattern into 0’s and 1’s. This binarized pattern, shown in Fig. 6(a), will then be used to produce the computer-aided design (CAD) file that nanofabricates the holographic optical element. To see the effect of binarization on the reconstructed wavefront, this binarized holographic optical element is Fresnel propagated to the plane where the object wave converges to two points and the intensity distribution is shown in Fig. 6(b).

The CGH can be optimized for optical flux throughput, while the XOR pattern is not specifically designed for maximum efficiency. However, it is very difficult for the CAD program of an electron-beam column to generated a CGH data file due to the large memory requirement imposed by the large amount of very small and irregularly-shaped structures particularly at the outer edge of the CGH. In addition, the finer details required by the CGH also make it more difficult to nanofabricate. The XOR pattern provides a more practical solution in that it requires much less computer memory and relatively less stringency in nanofabrication. For the XOR pattern the digital data files of the grating and the zoneplate are already accurately calculated and taking the bit-wise XOR operation of the two data files is trivial in computers.

7. Conclusion

To the best of our knowledge, this paper demonstrates, for the first time, a novel diffractive optical element based on Fourier optics techniques. It is shown, both in theory and in experiment, that by combining two diffractive elements, a grating and a zoneplate, through a bit-wise XOR operation, the resultant optical element produced a new functionality: two symmetric off-axis foci with a higher efficiency. The two symmetric off-axis foci at the back focal plane are ideal for interferometric experiments. Specifically, it is shown that interferometric experiments that require better contrast and higher coherent power benefit from this XOR design due to the symmetricalness of the intensity distribution at the back focal plane and the improved overall efficiency, respectively. Although useful at all wavelengths, this pattern has particular value at the short wavelengths of interest here. This group of optical elements shown in this paper brings sophisticated Fourier optical techniques to open new experimental frontiers in an area rich with opportunities on nanometer scales and with element-specific identifications and applications.

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