Physics of Planar X-Ray Waveguide

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A simple model of X-Ray standing waves (XSW) formation in the slit between two polished parallel flat plates formed a planar X-Ray waveguide for the angle area restricted by the critical total reflection angle is developed. It is shown that the model is true for a case of the Bragg reflection. The conditions required for XSW to appear in the slit space are formulated and a slit size interval conforming to these conditions is evaluated. A mechanism of a XSW intensity decrease in a planar X-Ray waveguide is proposed. The efficiency of the waveguide on a comparison base of the X-Ray forming system completed by a slit-cut superposition set with one equiped by a planar X-Ray waveguide is evaluated.

Some recommendations on the application of the planar X-Ray waveguide in X-Ray structural and spectral studies of surface are presented.

Key words: total reflection, X-Ray standing wave (XSW), planar X-Ray waveguide, planar X-Ray waveguide-monochromator

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**Introduction**

Optical and infrared waveguides are conventional elements of the most optical instruments today. An elaboration of similar elements for the short lengthwave range is long ago the sacred dream of many researchers [1, 2, 3]. The ideal medium for a hard electromagnetic radiation waveguide core is vacuum which has the highest possible refractive index \( n = 1 \) for X- and \( \gamma \)-rays and no absorption losses. But all researches in field of such waveguide formation were connected with a real materials as a core of them, such as BN, C, Be, polyimide, characterized by non zero absorptions. The introduction of hard electromagnetic radiation into a waveguide film for the multilayer waveconductor structure was carried out by the optical type injection - by means of resonant beam couplers. This attempts of the waveguide building on base of the multilayer media using had not led yet to success but they obtained some interesting results. For example, authors of the work [2] observed 20-fold flux increasing which was agreed with them theoretical predictions and was limited by absorbtion of an X-Ray radiation with energy \( E \approx 12 \text{ keV} \) in the layer’s media, only.

At the same time the other direction been related with the X-Ray beam transportation had been developed. It was connected with the slitless collimator application for TXRF analysis [4, 5, 6, 7]. The slitless collimator applied in those works was formed by two quartz polished plates mated together. The clearance in it was formed due to roughness and microsphericity of the plates. Experiments showed, that these slitless collimators could transport X-Ray radiation over a distance of 100 mm without visible intensity decrease. The simplest model would had been able to explain beautiful transportation properties of the slitless collimator for an X-Ray radiation propagation is the model of a multiple X-Ray beam total reflection in the clearance between plate polished reflectors [8]. Results of such model application for a description of the X-Ray beam moving in the clearance of a slitless collimator is represented in the next section.

**Model of a multiple total reflection in an X-Ray slitless collimator clearance**

Slitless collimator (Fig. 1a) is the conventional name because the X-Ray beam is passed through the clearance between the plates but the slit is not observed by a visual perception. The size of the clearance is not permanent
parameter and is connected with reflectors surface nonperfections. The application of an attenuated total reflection method for the plate’s clearance diagnostic shows that the average width of the microslit «d» is near 30 nm. In the multiple reflection model the X-Ray beam propagates through the microslit by way of repeated total reflections between flats of the plates. The Fresnel coefficients for the single X-Ray beam reflection can be written as [9]:

\[
R^{(1)}_\parallel = \frac{|E^R_\parallel|}{|E^I_\parallel|^2} = \frac{(\theta-2\delta-a)^2+(b+2\beta\theta)^2}{(\theta-2\delta+a)^2+(b+2\beta\theta)^2}
\]

\[
R^{(1)}_\perp = \frac{|E^R_\perp|}{|E^R|^2} = \frac{(\theta-a)^2+b^2}{(\theta+a)^2+b^2}
\]

Expressions for «a» and «b» have the next forms:

\[
a^2 = \frac{1}{2} \left[ \sqrt{(\theta^2 - 2\delta)^2 + 4\beta^2} + (\theta^2 - 2\delta) \right]
\]

\[
b^2 = \frac{1}{2} \left[ \sqrt{(\theta^2 - 2\delta)^2 + 4\beta^2} - (\theta^2 - 2\delta) \right]
\]

where \(\theta\) is the incident angle of the beam, \(\delta\) and \(\beta\) are parameters of the complex addition to the matter refraction index \(n\) in the form:

\[
n = 1 - \delta - i\beta
\]

The real parameter \(\delta\) is concerned with the critical incident angle \(\theta_c\):

\[
\theta_c = \sqrt{2\delta} = \sqrt{8.24 \cdot 10^{-4} \frac{Z}{A} \frac{\rho}{E^2}}
\]

where \(\theta_c\) is represented in radians, \(\rho\) is the material density (g/cm\(^3\)), \(E\) is the radiation energy (keV), \(Z\) is the target element atomic number, \(A\) is the atomic weight (\(Z\) and \(A\) are dimensionless values). The imaginary parameter \(\beta\) is related to linear X-Ray absorption coefficient of the material:

\[
\tau = \frac{4\pi}{\lambda} \beta
\]

where \(\lambda\) is the wavelength of the incident radiation. Figure 1b shows dependence of reflection factors \(R^{(N)}_\parallel\) and \(R^{(N)}_\perp\) for single and multiple X-Ray beam reflections from a quartz plate (\(\delta_{SiO_2} = 1.6 \cdot 10^{-6}, \beta_{SiO_2} = 0.4 \cdot 10^{-8}\)). From this figure we notice that the X-Ray beam after reflection is not characterized by any preferred direction of the polarization. The divergence of the X-Ray beam suffered multiple reflections in the clearance of a slit (Fig.
is restricted by the angle value $\Delta \theta = 2\theta_c$. The integral factor of the reflection for the fan-shaping beam after one hundred reflections must fall down to 0.7, approximately. In light of the such calculations, it is needed to evaluate the real number of reflections taking place in an X-Ray beam propagation across the slitless collimator of real TXRF spectrometer.

X-Ray beam reflection for angles $\theta \leq \theta_c$ in the plate’s clearance of the slitless collimator is connected with the penetration into the reflector surface and a displacement along the clearance. The depth penetration of X-Ray radiation into the material in condition of the total reflection may be represented by expression [9]:

$$x_e(\theta) = \frac{1}{2\sqrt{2\pi}} \cdot \frac{\lambda}{\sqrt{(\theta_c^2 - \theta^2)^2 + \beta^2 + (\theta_c^2 - \theta^2)}}$$  \hspace{1cm} (6)

The depth penetration is the function of the X-Ray incident angle. To evacuate the real number of reflections in the clearance, the effective $x_e(\theta)$ must be calculated. As the slitless collimator transports X-Ray partial beams with reflection angles from the interval $0 \leq \theta \leq \theta_c$, the effective angle would be choose for the creation of the simplified model described the multireflection process of the X-Ray sector-shaped beam. As a such one, the half critical angle $\theta_c/2$ may be offered [6]. The effective depth penetration is corresponded to angle $\theta = \theta_c/2$.

The phenomenon of a wave total reflection is characterized by the Goos-Hanchen wave front displacement of the beam reflection position about the point of the beam’s incoming place [10, 11, 12]:

$$\Delta z = \frac{\lambda}{\pi} \frac{1}{\sqrt{\theta_c^2 - \theta^2 + 2i\beta}}$$  \hspace{1cm} (7)

Maximum displacement corresponds to $\theta = \theta_c$. The displacement module in that case is equal to:

$$|\Delta z_{max}| = \frac{\lambda}{\pi} \frac{1}{\sqrt{2\beta}}$$  \hspace{1cm} (8)

Minimum displacement is arrived at $\theta = 0$ and has the value:

$$\Delta z_{min} = \frac{\lambda}{\pi\theta}$$  \hspace{1cm} (9)

Effective value $\Delta z_e$ may be calculated by using the reflection angle $\theta_{cf} = \theta_c/2$. The effective displacement $\Delta z_{ef}$ of the total reflection point on the quartz reflector for the MoK$_\alpha$ radiation is equal to 146 Å and the displacement interval stretches from 128 Å to 2500 Å. Knowledge of $\Delta z_{ef}$ value
and the size of the clearance permits the quantity \( (N) \) of beam reflections in the clearance:

\[
N = \frac{L}{\Delta z_{ef} + d \cdot \text{ctan}(\theta_c/2)}
\]  

where \( L \) is a length of the collimator, \( d \) is the size of the clearance. For real collimator \((L = 5 \text{ cm}; \ d = 30 \text{ nm})\) \( N \) is approximately equal to 3000. The comparison of the X-Ray beam intensity calculated from the multireflection model \((\text{Fig. 1})\) \( I_{\text{mm}}(\theta_c/2) = 0.0003I_0 \) with real intensity at the output of the slitless collimator \([4, 6]\) \( I_{\text{SC}} \approx 0.9I_0 \) shows that the model is unsuitable for a description of the X-Ray beam transportation by the slitless collimator. This contradiction assumes that the slitless collimator transports the X-Ray beam by creating a standing wave in the clearance – forming a standing wave channel.

**Formation of an X-Ray standing wave upon specular reflection of a plane wave**

Assume that an electromagnetic monochromatic plane travelling wave with the \( \sigma \)-polarization \((\text{i.e. } \vec{E}_0 \text{ perpendicular to the } x-z \text{ plane in Fig. 2})\), wavelength \( \lambda_0 \) and wave vector \( k_0 = \frac{1}{\lambda_0} \) impinges on the boundary separating two materials. If the materials have different refraction indices, part of the wave energy is reflected and the remainder passes to the second material or is refracted. An interference field appears in the first material irrespective of the remainder value. The interference field area depends on the width of an incident plane wave and the incidence angle \( \theta \). The intensity of the interference area is directly determined by the reflection factor on the boundary between materials and peaks at the total reflection of an incident radiation beam. Referring to Fig. 2, the incident and reflected travelling E-field plane waves can be described as \([13]\):

\[
\vec{A}_0(\vec{r}; t) = \vec{E}_0 e^{i[\omega t - 2\pi(k_x x - k_z z)]}
\]

and

\[
\vec{A}_R(\vec{r}; t) = \vec{E}_R e^{i[\omega t - 2\pi(k_x x + k_z z)]}
\]

For the sake of convenience, let \( z = 0 \) corresponds to the reflector surface.
By locating the intersections of the crests and troughs of the two travelling plane waves in Fig. 2, one can easily show that the interference between the two coherent waves generates a standing wave with planes of maximum and minimum intensity parallel to the boundary surface. The period of the standing wave is defined by expression [14]:

\[ D = \frac{\lambda}{2 \cdot \sin \theta} \]  

(13)

In a general case the amplitude relation between the incident and reflected waves is described by the Fresnel equations [13]:

\[ \left| \frac{E_R}{E_0} \right|_\perp = \frac{\sin \theta - n \sin \varphi}{\sin \theta + n \sin \varphi} \]  

(14)

where \( \varphi \) is the refraction angle and \( n \) is the relative refraction index. The phase discrepancy for the electric vector in the incident and reflected waves \( \psi \) for the total reflection phenomenon is defined by expression:

\[ \tan \frac{\psi}{2} = \frac{\sqrt{\cos^2 \theta - n^2}}{\sin \theta} \]  

(15)

Equations (13), (14) and (15) describe the reflection phenomenon for any electromagnetic plane wave on a plane interface between two materials. They can also be used to describe, as a first approximation, a total external X-Ray reflection on a vacuum - material interface, which is usually represented as the specular reflection of an X-Ray plane wave, for the sake of simplification².

In the conjunction of the secular reflection for an X-Ray radiation the amplitude relation between electric vectors of the incident and reflected waves and the phase expression have the from [9, 14, 15]:

\[ \left| \frac{E_R}{E_0} \right|_\perp = \frac{\theta - \sqrt{\theta^2 - 2\delta - 2i\beta}}{\theta + \sqrt{\theta^2 - 2\delta - 2i\beta}} \]  

(16)

and

\[ \cos \psi = \begin{cases} 2 \left( \frac{\theta}{\theta_c} \right)^2 - 1 & \text{for } \theta \leq \theta_c \\ 1 & \text{for } \theta > \theta_c \end{cases} \]  

(17)

²This simplified representation disregards the Goos-Hanchen wave front displacement at total wave reflection [10], and the radiation penetration to top layers of the material plate.
The X-Ray radiation intensity in the vacuum over the boundary surface in the interference field area usually defined as $|\vec{A}_0 + \vec{A}_R|^2$ can be presented by the next formula [14]:

$$ I (\theta, z) = |\vec{E}_0|^2 \left[ 1 + R + 2\sqrt{R} \cos \left( \psi - \frac{2\pi z}{D} \right) \right] $$

where $D$ is the standing wave period along the z-coordinate, expressed by Eqn.(13) and $R$ is the reflectivity factor been calculated by Eqn.(1):

The interference field can be observed both at $\theta < \theta_c$ and $\theta > \theta_c$, but in the latter angle range its intensity decreases abruptly [15]. The standing wave period $D$ achieves its minimum at $\theta = \theta_c$ in the $0 \leq \theta \leq \theta_c$ range. As the incident angle decreases, the period $D$ increases to become infinitely large at the grazing incident angle ($\theta = 0$). But this is not case in practice, because the coherence of incidence and reflected beams is broken. The most obvious factor causing the interference field erosion is the width finiteness of incident radiation lines $\Delta \lambda$ [13]. It is generally accepted, that the interference field does not become smeared, if the condition [9]:

$$ \Delta \lambda \leq \frac{\lambda}{4} $$

holds.

Another factor influencing the interference field picture is the roughness of a reflecting surface. Evaluations show that the interference field does not undergo smearing at $\theta = \theta_c$ when the height of microheterogeneities on the reflected surface does not exceed the critical size $h_c$ [9]:

$$ h_c = \frac{\lambda}{8\sqrt{2\delta}} = \frac{1}{8} \sqrt{\frac{\pi mc^2 A}{e^2 N \rho Z'}} $$

The critical roughness parameter does not depend on the wavelength of the incident radiation. Its magnitude for polished optical quartz plates is equal to 5 nm.

**Standing wave formation in the slit of an X-Ray planar waveguide**

A successive reflection of a plane electromagnetic wave in the slit formed by two parallel plates results in the formation of several interference field areas in it (Fig. 3a). Varying the slit size can lead to overlapping the areas to
create the uniform interference field zone within the total space of the slit (Fig. 3b\(^3\)). So, it can be expected that an XSW excitation can be formed in the plane slit, when its width falls within a certain range. The minimum slit size (secular minimum size) for this can be evaluated from expression (13) for the critical angle of a total reflection:

\[
D_{\text{min}} = \frac{\lambda}{2 \sin \theta_c} \approx \frac{\lambda}{2 \theta_c} = \sqrt{\frac{\pi mc^2 A}{2e^2 NZ' \rho}}
\]  

(21)

The minimum slit size promoting the XSW formation is independent of an X-Ray incident radiation wavelength. The material structure density \(\rho\) of reflectors is a real factor influencing the minimum size \(D_{\text{min}}\). Its magnitude for quartz reflectors \((\bar{Z} = 10; \bar{A} = 20)\) is 21 nm. In practice the \(D_{\text{min}}^*\) value is smaller because an XSW is characterized by a visible intensity up to the X-Ray penetration depth. The effective value of the penetration depth parameter for an X-Ray slitless collimator is 3.6 nm for MoK\(_\alpha\) radiation impinging on a quartz plate. So, the real size of the minimum clearance between the quartz plates \(D_{\text{min}}^*\) is found to be approximately 14 nm. This value is comparable sized with the double roughness of reflectors 10 nm.

The upper restriction for the slit in an X-Ray waveguide can be evaluated using ratio Eqn.(19):

\[
D_{\text{max}} = \frac{\lambda}{4 \left(\frac{\Delta \lambda}{\lambda}\right)} = \frac{\lambda^2}{4 \Delta \lambda}
\]

(22)

For MoK\(_\alpha\) radiation, the wavelength is \(\lambda = 0.707 \times 10^{-1}\) nm and \(\Delta \lambda = 0.29 \times 10^{-4}\) nm [16]. Substituting these values into Eqn.(22) gives \(D_{\text{max}} = 43\) nm. In practice, its magnitude is \(D_{\text{max}}^* = 36\) nm. Similar value for CuK\(_\alpha\) radiation is equal to \(D_{\text{max}}^* = 91\) nm. Analogous parameters calculated for the set of X-Ray and gamma radiation are collected in Table 1. Quartz total reflection parameters peculiar to the radiation set are represented in the same place. It should be stressed that the X-Ray spectra in incident and emergent beams do not agree between them. For example, if we shall try to form

\(^3\)It is very important to notice that the uniform interference field zone will be appear, in the context of the specular reflection model, for the some specific reflection angles, only. The uniform interference field zone appearance for any reflection angle can be obtained by taking into a consideration the expansion of the interference field into a layers top of reflectors and the Goos-Hanchen displacement.

\(^4\)The important practical parameter influencing on the upper slit size is a degree of the plate parallelity in an X-Ray waveguide.
the X-Ray beam excited by X-Ray tube with Mo anode by using of an X-Ray waveguide with a slit size \( D > D_{\text{max}} \), we shall not find the characteristic deposit in the spectrum of an emergent beam. But if the size will be not excel the value \( D_{\text{max}} \) (\( D_{\text{min}} < D < D_{\text{max}} \)), the characteristic radiation MoK\(_\alpha\) will be present in the emergent beam X-Ray spectrum with a great intensity. It can be expected that the slit width varying in a waveguide would cause the X-Ray spectrum modification in the emergent beam at the transportation of the white X-Ray radiation.

Minimum slit sizes presented in Table 1 (without correction on the X-Ray depth penetration) are the constant. Maximum values of this parameter are near 100 nm for the K\( \alpha \) radiation of Fe group elements. \( D_{\text{max}} \) and \( D_{\text{min}} \) magnitudes approach each other for the hard X-Ray radiation (MoK\(_\alpha\), AgK\(_\alpha\)). The greatest values of \( D_{\text{max}} \) for the gamma radiation have engaged our attention primarily. A significant difference between \( D_{\text{max}} \) values for X-Ray and nuclear radiations presents a unique possibility for its separation in complicate spectra, even though the radiation wavelengths coinside.

### Attenuation of an X-Ray standing wave in a planar slit waveguide

The excitation of a standing wave in a waveguide slit gives rise to a stationary distribution of the interference field intensity both along the slit channel (along axis \( x \)) and crosswise (along axis \( z \)). This distribution is pictured in Fig. 4 to fit the input of a waveguide \( (z = 0) \). The distribution is plotted for a plane X-Ray beam (CuK\(_\alpha\)) impinging into a slit of the quartz waveguide under an angle \( \theta = 0.92 \cdot \theta_c \) on the reflector surface. The reflection conditions correspond to a value phase variation \( \psi \approx 45^\circ \) and a standing wavelength \( D \approx 1.1 \cdot D_{\text{min}} \). The arising standing wave is characterized by the penetration depth \( z_e = 8.6 \) nm. Hence, the distribution shown in Fig. 4 obeys the expression \( z_e \approx 0.4 \cdot D_{\text{min}} \).

The standing wave intensity within the slit is described by expression (18). Outside the slit, the intensity decreases with decrement \( \frac{1}{z_e} \):

\[
J(z) = I(\theta_c; z) e^{-z/z_e}
\]

where \( I(\theta_c; z) \) is the undisturbed function of a standing wave intensity defined by expression (18). Integration of Eqn.(18) and Eqn.(23) produces the total intensity for a standing wave in the cross-section of an X-Ray waveguide. The domain of integration for expression (18) is equal to a slit size. The standing wave intensity distribution is shown in Fig. 4.
wave total intensity in the reflector top layers can be calculated, to a first approximation, by integrating of function Eqn.(23) in the top layer domain $1.5 \cdot D_{\text{min}}$ for each reflector. Calculations for the CuK$_\alpha$ radiation in the quartz waveguide show that the total intensity concentrated in the slit is equal to $L(0) \sim 9.2 E_0^2 D_{\text{min}}$ and the total intensity connected with top layers of the reflectors is $M(0) \sim 1.2 \cdot E_0^2 D_{\text{min}}$. The standing wave propagation along the slit channel of a waveguide is characterized by retaining the energy relation between a slit and top layers of the reflectors. It means that the relation between the energy in top layers and the total energy of a standing wave holds too:

$$\alpha (x) = \frac{M (x)}{L (x) + M (x)} = \text{const}$$

Because attenuation of a standing wave occurs only by the absorbion in reflector top layers, equality (24) implies continuous energy transfer between different standing wave parts. The attenuation of a standing wave intensity can then be described by the expression:

$$W (x) = [L (0) + M (0)] e^{-\alpha x} = W(0) \cdot e^{-\alpha x}$$

(25)

The magnitude of $\alpha$ depends on the wavelength of incident radiation, the reflector material properties, the angle of radiation impinging and the width of a waveguide slit. The $\alpha$ dependence from the incident angle can be evaluated by calculating its variation with some incident angles area of CuK$_\alpha$ radiation in the quartz waveguide. This gives the values: $\alpha(\theta_c) = 0.8; \alpha(\theta_c/2) = 0.05$. The value of $\alpha$ decreases abruptly, if the slit width exceeds the wavelength of a standing wave.

Using formula (25), the values of $\alpha$ for the incident angle $\theta = 0.92 \theta_c$ and the condition $s_{\text{slit}} \approx 4D$, we one can calculate the total intensity attenuation for a standing wave of CuK$_\alpha$ radiation in the planar quartz extensible waveguide. The total intensity of a standing wave after passing the way $\Delta x = 10$ mm in the waveguide is $W(\theta_c) = 0.3 W_0$ and $W(\theta_c/2) = 0.6 W_0$. Note that the model of multiple total successive reflection under similar conditions gives: $W_0 \cdot 10^{-27}$ and $0.006 W_0$, respectively.

The calculated data for the total intensity attenuation of an XSW in a waveguide lend an explanation of the high efficiency of X-Ray slitless collimators for X-Ray radiation transport over long distances [4, 5, 6, 7]. However, the evaluations of clearance sizes between the mated quartz plates in those works strongly disagree ($D \approx 30$ nm [7], $D \approx 150$ nm [4]). Moreover, the slit width in the collimators is not a stable parameter and can vary along the collimator length. Therefore, the evaluations of quantity parameters of
the emergent beam intensities for a slitless collimator must be treated with caution. In addition, note that the slit width variation can bring about a considerable modification of the X-Ray spectrum of an emergent beam, if the initial X-Ray radiation is a mixed type (white and characteristics).

The above results help to elucidate the advantages and shortcomings of X-Ray slitless collimators. Moreover, they can become the basis for an X-Ray waveguide designing with properties predicted and high intensity of an emergent beam. The experimental part of the work is devoted to investigation of a real planar X-Ray waveguide with the clearance size \( d \leq D_{\text{max}} \) for the quartz reflector plates and CuK\(_{\alpha}\) radiation.

**Experimental setup**

The experimental part of the work was carried out with using the modified horizontal type diffractometer HZG-4 Karl Zeiss Jena production. The modification of the diffractometer was connected with an elaboration of a specific equipment for the work with waveguide devices and the carrying out of measurements with a diffractometer radius up to 400 mm. The waveguide used in the work has the length \( \ell = 100 \) mm, the size of a slit (clearance) \( d = 88 \) nm between polished quartz reflectors. The initial X-Ray beam is not collimated in vertical direction by a Soller slit block because the height of a waveguide slit is equal to 10 mm. The vacuum tube with immovable copper anode and upper limit power 2 kW was used as an X-Ray radiation source. The real size of a line focal spot projection was 0.2 mm. The intensities as an attenuated direct X-Ray beam and a diffracted one were registrated by a scintillation counter supplied by Russian photomultiplier tube FEU-85 and scintillation crystal NaI(Tl) with diameter \( D = 20 \) mm and thickness \( t = 0.1 \) mm. The using of a thin scintillation crystal allows together with an amplitude discriminator application to suppress an X-Ray "white" radiation deposit in a detected signal. The analogue treatment of the signal was carried out by pulse electronics blocks in NIM standard produced by Ortec firm. Diffractometer patterns and X-Ray spectrograms were collected in analogue form by the recorder K-100 type Karl Zeiss Jena production or digital form by Quad Counte/Timer Ortec 872. The attenuation of a direct X-Ray beam realized by Cu-film absorber with thickness \( t = 0.1 \) mm characterized by a attenuation factor \( K = 207 \) for the characteristic CuK\(_{\alpha}\) radiation.
Experimental study of planar X-Ray waveguide properties

The efficiency application of PXW as an X-Ray beam formation device was studied on base of a comparative analysis of the beam after a conventional slit system and a waveguide collimation. The position of PXW in the X-Ray optical arrangement of the diffractometer is chosen from conditions of an aligned convenience because the aligning procedure for the instrument contained a device with a slit width $d = 88$ nm is not trivial task. The geometrical schemes of those arrangements are shown on a top part of Fig. 5. A conventional arrangement uses a single slit-cut system. Experiments are carried out with three sizes of the slit: 0.1 mm; 0.25 mm and 0.5 mm. The distance between a line focus projection and the slit of a detector collimator was equal near 700 mm. The size of the detector collimator slit was chosen 0.1 mm. The angle size of the slit is corresponded to $\delta \omega = 0.01^\circ$. In researches been devoted to the analysis of an X-Ray beam formed by a slit system or a waveguide the amplitude discriminator was tuned on a transmitting of the pulses corresponded to the characteristic radiation CuK$_{\alpha\beta}$, only.

Outlines described X-Ray intensity distribution been typical for the slit system of an X-Ray beam formation are shown in bottom part of Fig. 5. The outline corresponded to the widest formation slit ($d = 0.5$ mm) permits to evaluate the real intensity of the characteristic radiation portion in the beam generated by our X-Ray tube at $U = 20$ keV and $I = 10$ mA. Taking into account a detector collimator slit area (1 mm$^2$) and a attenuation factor of a absorber one can calculate the total intensity of a characteristic part in the beam for the detector position point $J = 2 \cdot 10^7$ quantum/cm$^2$sec. The optimal divergence of the beam can be evaluated as $\Delta \theta = 0.21^\circ$ (slit size $d = 0.25$ mm). Form of outlines is closed to a triangular one.

The outline for the X-Ray characteristic beam formed by PXW ($\ell = 100$ mm; $d = 88$ nm) displaced on a distance $\ell_1 = 75$ mm about a focus spot is shown in bottom part of Fig. 5. The total intensity been typical for this distribution is approximately corresponded to an intensity of the beam formed by the slit system. The real value of a beam divergence for the chosen geometry was equal $\Delta \theta = 0.18^\circ$. The outline of an angle intensity distribution for the X-Ray beam formed by PXW is described by Gaussian form. The form modification of the outline for PXW case is explained by an existence of an X-Ray radiation penetration into top layers of waveguide reflectors flats.

The density evaluation for an X-Ray beam inside of a waveguide slit may
be carried out by a comparison of the total intensity an X-Ray beam formed by waveguide and slit-cut collimation systems. Calculations show that a total intensity of the beam formed by PXW is equal to one of the beam formed by a conventional system equipped by a single slit with size $d = 0.15$ mm. By this is meant that an X-Ray radiation density inside the waveguide slit and near its outlet is in excess of $2 \cdot 10^3$ the density of the X-Ray beam formed by a slit system for those space positions. Thus, one can expect that the waveguide application will be very efficient device for the analysis of a small objects.

The total intensity obtained in our experiments for the waveguide used was not the greatest possible for the chosen X-Ray tube regime. The maximum intensity can be achieved at the optimal waveguide positioning about the focal spot of an X-Ray tube. Such disposition will be corresponded to a waveguide X-Ray capture angle $\Delta \theta = 2\theta_c$ ($\Delta \theta = 0.42^\circ$ for CuK$_\alpha$ and SiO$_2$ reflectors) and a distance between the focal spot and the waveguide inlet $\ell_1 = 25$ mm. The disposition optimization will lead to the total intensity increasing in 20 times in comparison with the chosen geometry. But in this case the beam divergence will increase too and achieve the value $\Delta \theta = 2\theta_c$. The procedure of the beam divergence decreasing without falling of its total intensity is the technical problem and will be described in details in other work.

The energy spectrum of the beam formed by PXW is a prime interest for both the practical purposes and the understanding of an X-Ray waveguide physics. The spectrum of an energy distribution was analyzed by using of the modified diffractometer HZG-4 and a perfect monocrystal SiO$_2$ (101) as a desperger in the angle interval $2^\circ \leq \theta \leq 20^\circ$. Diffractometer patterns of an X-Ray energy dispersion for the beam formed by a planar X-Ray waveguide obtained at several tube regimes are shown in Fig. 6. The geometry of measurements sketches on an insertion. Since the angle interval chosen contains single reflection set of the crystal-disperger (101), this area can be used for a dispersion analysis of the energy spectrum for the emergent waveguide beam. This measurements were carried out at the open discriminator window. Vertical axis represents a reflection intensity. It is particularly remarkable that the real intensity for the peak (101) CuK$_\alpha$, in the geometry chosen enhanced 100,000 counts per second. Horizontal axis represents the dispersion angle scale together with scales for the wavelength and the energy for an radiation analyzed. Diffractometer patterns (energy diagrams) shows that the intensity of a "white" X-Ray radiation in the spectrum of an emergent waveguide beam is not high, although one can notice some intensity increasing in the high edge of the energy area.

Data offered in Fig. 6 can be considered as the indirect evidence of a
justice for the standing wave conception explained a high efficiency of the waveguide ability to transport of an X-Radiation without losses, only. The waveguide slit width \( d = 88 \text{ nm} \) was chosen on base of calculations carried out in work [17] under beautiful transmission of the characteristic radiation CuK\(_{\alpha,\beta} \). The conception of a standing wave formation in a waveguide clearance asserts that the slit size \( d = 88 \text{ nm} \) assists to a standing wave excitation for the X-Ray energy region \( E < 9 \text{ keV} \). This limit is connected with the parameter of a radiation length coherence. The characteristic radiation with a certain value of the length coherence generates the standing wave in a waveguide clearance. But one can not notice the step on the intensity overline in the region of \( E \approx 9 \text{ keV} \). This is to say that a white radiation is characterized by a abruptly distinguishing length coherence or the calculation carried out in [17] is inexact. This discrepancy will be get over after complication of our works with the waveguides set characterized by various slit widths. The increasing of a diffraction intensity for the high energy X-Ray radiation, presumably, can be connected with mechanism manifestation of the multiple X-Ray total reflection [8] because the attenuation of a short wave radiation is smaller as a long wave one.

The comparison with spectra of the beams formed by a conventional slit system was not carried out because the our scintillation detector is equipped by thin scintillation crystal registrated high energy radiation with lower efficiencies.

**Planar X-Ray Waveguide – Monochromator**

Preceding sections are devoted to discussion of the planar X-Ray waveguide using the total X-Ray reflection phenomenon for the standing wave generation in a slit space of the device. But it is well known that the excitation of X-Ray standing wave is possible for the Bragg geometry too [18]. The X-Ray standing wave arises in a slit space between two parallel polished reflectors, if the reflectors are perfect monocrystal been orientated mutually. The mechanism of the standing wave formation in Bragg reflection conditions is not practically differed from one in the case of a total X-Ray reflection (Fig. 2,3). But a formal consideration of a standing wave formation in Bragg planar X-Ray waveguide (waveguide-monochromator) requests replacement of the X-Ray depth penetration parameter \( z_e \) on the parameter of the primary extinction length \( z_{ext} \) [19]:

\[
    z_{ext} = \frac{1}{\sigma} = \frac{\sin \theta_b}{2\lambda |c|} \cdot \frac{mc^2}{e^2} \cdot \frac{\nu}{|F_h|} \tag{26}
\]
where $\sigma$ is the extinction factor, $c$ is the polarization factor been equal to unity for a $\sigma$-polarization, $c$ is the light velocity, $m$ and $e$ are the electron characteristics, $v$ is the unit cell volume for the reflector material, $F_h$ is the relative structure factor of the chosen reflection. The $z_{ext}$ magnitude defines the crystal thickness attenuated of the X-Ray beam intensity falling on the crystal under Bragg angle with $<e>$ factor. It is significantly that the primary extinction length is not depended on a wavelength of an X-Ray radiation. Magnitudes of the primary extinction length are usually two orders higher as values of the depth penetration at the X-Ray total reflection. For example, the magnitude of $z_{ext}$ for (200) NaCl reflection is equal to 660 nm [19]. Because of this, the practical size of a waveguide-monochromator slit may be visibly differed from one calculated and its efficiency must be lower as compared to a waveguide built on the total reflection phenomenon. But the X-Ray radiation density in its emergent beam will be significantly higher as one for the conventional Bonse-Hart monochromator [20]. The planar X-Ray waveguide-monochromator functions is conceptually identical with the Borrmann effect manifestation in perfect crystals [21, 22]. Authors hope to give a comprehensive analysis of peculiarities which are typical for the planar X-Ray waveguide-monochromator in other work.

**Application aspects of a planar X-Ray radiation waveguide**

An X-Ray slitless collimator is a planar X-Ray waveguide with an uncontrolled size of the waveguide slit, which, can be varied during one experiment. Although these variations are not great, an X-Ray slitless collimator should be regarded as a simple and convenient experimental model. For practical purposes, slit waveguides are needed with the greatest possible slit size for a chosen wavelength. Quartz waveguides with slit sizes 36 nm and 91 nm are best suited for MoK$_\alpha$ and CuK$_\alpha$ radiations, respectively. To manufacture such waveguides, the metal thin strips are deposited on to edges of one quartz reflector of a waveguide, and then the waveguide is uniformly compressed between metal plates. The clearance magnitude of a slit can be controlled by the Optical Attenuated Total Reflection method [23].

An X-Ray slitless collimator came into use for X-Ray fluorescence analysis of plane surfaces and thin films (TXRF-SC analysis) some years ago [6]. The substitution of the slitless collimator by an X-Ray waveguide considerably increases X-Ray radiation density on the surface of a target analyzed.
and ensures the diagnostic reproduction. Furthermore, an X-Ray waveguide helps avoid the target contact with reflector surfaces. Otherwise, an X-Ray planar waveguide retains all advantages of a slitless collimator in TXRF spectroscopy. But it is notice that the spurious peaks do not disappear at the waveguide application.

Another important field of a planar waveguide possible using is the X-Ray diffraction researches. It can be used for structure investigation of monocrystal surfaces and epitaxial films at the total reflection of an incident X-Ray beam in the parallel and perpendicular geometries [24] because a waveguide ensures high X-Ray radiation density in the beam. The planar X-Ray waveguide holds greatest promise for using in commercial diffractometers for symmetrical and asymmetrical geometries, too.

**Conclusion**

The model of XSW excitation in a planar slit waveguide has been developed by employing the interference wave theory to treat areas whose size considerably exceeds the wavelength of initial radiation but the coherence conditions are still valid. An example of practical embodiment of the idea of such a waveguide is an X-Ray slitless collimator whose unique properties could only be explained in terms of the model of XSW excitation. The evaluation of the upper and lower boundaries for the slit width which provide XSW excitation, points out the way to waveguides building with most efficient for particular purposes.

The presented model is a simple one and disregards some phenomena: the Goos-Hanchen effect and monotonous variation of the refraction index on a vacuum-material interface [25]. However, even the simplified evaluations made in the work show that a planar X-Ray waveguide and a waveguide-monochromator can became useful tools for the X-Ray diffraction and spectroscopic investigations, especially for the work with synchrotron radiation.

**It is very important that the model can be applied for the neutron and electron beams and will stimulate the waveguides creation both for the white radiation and for the monochromatic one for them.** The waveguide-monochromator can be basis for the building of a laser pumping system applikated for the hard X-Ray and gamma radiation.
Acknowledgements

The authors thank d-r A.V. Okhulkov for useful discussion of the problems and O.S. Kondratiev for the help in calculations.

Reference


Table 1. Slit size values \(D_{\text{min}}^{(1)}\) and \(D_{\text{max}}^{(1)}\) allowing an X-Ray standing wave excitation in the clearance of planar X-Ray waveguide with quartz reflectors for the set of an X-Ray and gamma radiation are collected. Some attendant parameters are presented too.

<table>
<thead>
<tr>
<th>Radiation</th>
<th>(E_0) (keV)</th>
<th>(\lambda_0) (nm)</th>
<th>(\Delta \lambda) (nm)</th>
<th>(D_{\text{min}}) (nm)</th>
<th>(D_{\text{max}}) (nm)</th>
<th>(\theta_c)</th>
<th>(x_c(\theta_c)) (nm)</th>
<th>(x_c(\theta_{\frac{\pi}{2}})) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlK(\alpha)</td>
<td>1.486</td>
<td>0.8337</td>
<td>4.1(\times)10(^{-4}) (^{(2)})</td>
<td>21</td>
<td>424</td>
<td>1.16°</td>
<td>35.9</td>
<td>3.8</td>
</tr>
<tr>
<td>SiK(\alpha_1)</td>
<td>1.740</td>
<td>0.7135</td>
<td>3.5(\times)10(^{-4}) (^{(2)})</td>
<td>21</td>
<td>364</td>
<td>0.99°</td>
<td>37.9</td>
<td>3.8</td>
</tr>
<tr>
<td>CaK(\alpha_1)</td>
<td>3.691</td>
<td>0.3358</td>
<td>1.6(\times)10(^{-4})</td>
<td>21</td>
<td>176</td>
<td>0.46°</td>
<td>20.0</td>
<td>3.8</td>
</tr>
<tr>
<td>CrK(\alpha_1)</td>
<td>5.414</td>
<td>0.2290</td>
<td>1.03(\times)10(^{-4})</td>
<td>21</td>
<td>127</td>
<td>0.32°</td>
<td>27.9</td>
<td>3.8</td>
</tr>
<tr>
<td>FeK(\alpha_1)</td>
<td>6.403</td>
<td>0.1936</td>
<td>1.01(\times)10(^{-4})</td>
<td>21</td>
<td>93</td>
<td>0.27°</td>
<td>32.9</td>
<td>3.8</td>
</tr>
<tr>
<td>CoK(\alpha_1)</td>
<td>6.929</td>
<td>0.1789</td>
<td>8.1(\times)10(^{-4})</td>
<td>21</td>
<td>99</td>
<td>0.25°</td>
<td>35.2</td>
<td>3.8</td>
</tr>
<tr>
<td>CuK(\alpha_1)</td>
<td>8.046</td>
<td>0.1541</td>
<td>5.8(\times)10(^{-4})</td>
<td>21</td>
<td>102</td>
<td>0.21°</td>
<td>40.8</td>
<td>3.9</td>
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<tr>
<td>GeK(\alpha_1)</td>
<td>9.885</td>
<td>0.1254</td>
<td>4.3(\times)10(^{-4})</td>
<td>21</td>
<td>93</td>
<td>0.17°</td>
<td>48.4</td>
<td>3.9</td>
</tr>
<tr>
<td>(\gamma_1)Fe(^{57}) (\ast)</td>
<td>14.39</td>
<td>0.0862</td>
<td>0.28(\times)10(^{-10})</td>
<td>21</td>
<td>6.6(\times)10(^7)</td>
<td>0.12°</td>
<td>66.2</td>
<td>3.8</td>
</tr>
<tr>
<td>MoK(\alpha_1)</td>
<td>17.476</td>
<td>0.0709</td>
<td>0.29(\times)10(^{-4})</td>
<td>21</td>
<td>43</td>
<td>0.10°</td>
<td>83.1</td>
<td>3.7</td>
</tr>
<tr>
<td>AgK(\alpha_1)</td>
<td>22.159</td>
<td>0.0559</td>
<td>0.28(\times)10(^{-4})</td>
<td>21</td>
<td>21</td>
<td>0.077°</td>
<td>102.9</td>
<td>3.8</td>
</tr>
<tr>
<td>(\gamma_1)Sn(^{119}) (\ast)</td>
<td>23.80</td>
<td>0.0521</td>
<td>0.56(\times)10(^{-10})</td>
<td>21</td>
<td>1.2(\times)10(^7)</td>
<td>0.072°</td>
<td>110.4</td>
<td>3.8</td>
</tr>
</tbody>
</table>

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\(^1\)Parameters are calculated for a secular reflection approach.

\(^2\)The represented magnitudes are estimated.
Figure 1. (a) Scheme of a multiple X-Ray beam total reflection in the clearance of a slitless collimator. The way of an X-Ray beam is characterized by a penetration into the reflectors surface layers (Δz) and longitudinal displacement (Δx). (b) Dependencies of reflection factors $R_\parallel$ and $R_\perp$ from incident angle for the single reflection (1), ten consequent reflections (2), one hundred consequent reflections (3), thousand consequent reflections (4).
Figure 2. Classical scheme of the standing wave field formation at a specular reflection of a plane monochromatic wave above the mirror surface.
Figure 3. A scheme of the standing wave field formation at multiple successive nonspecular total reflections of a plane monochromatic wave (a), the principle scheme explaining the formation of the standing wave uniform zone upon trapping of a monochromatic plane wave by a plane extensive waveguide (b). $\Delta z$ - penetration depth of an X-Ray beam.
Figure 4. Intensity distribution function for an XSW in a waveguide slit and a top layer of quartz reflectors for an X-Ray beam impinging on the slit under a certain angle of total reflection $\theta$ for the quartz reflectors. The function without attenuation upon total reflection is shown by a dashed line. The function reflects the picture for $\lambda = 0.1541$ nm (CuK$_\alpha$); $\theta = 0.92 \cdot \theta_c$; $d = 97$ nm.
Figure 5. In the top of picture: sketches of geometries for an X-Ray beam form analysis at using of a convention slit-cut system (a) and after it passing of a planar X-Ray waveguide (b). Copper film with thickness $t=0.1$ mm was used as an X-Ray absorber-attenuator (filter) in both schemes.

In the foot of picture: a) Outlines of X-Ray beams formed by a conventional slit-cut system with a varying size of $S_1$ slit (Fig. 2a) 1: $S_1=0.5$ mm; 2: $S_1=0.25$ mm; 3: $S_1=0.1$ mm, $\ell_1=100$ mm, $\ell_2=600$ mm.

b) Outline of X-Ray beam, formed by PXW with $\ell=100$ mm; $d=88$ nm $\ell_1=75$ mm, $\ell_2=525$ mm. $S_2$ is equal to 0.1 mm for both geometries. Regime of an X-Ray tube (Cu anode) $U=20$ keV, $I=10$ mA.
Figure 6. Diffractometer patterns for monocrystal SiO₂ [101] obtained by HZG-4 precise X-Ray instrument equipped with a planar X-Ray waveguide $l_{PWX} = 100$ mm, $d = 88$ nm with the copper anode tube, 1: $U = 40$ keV, $I = 10$ mA; 2: $U = 30$ keV, $I = 10$ mA; 3: $U = 20$ keV, $I = 10$ mA. $l_1 = 75$ mm, $l_2 = 50$ mm, $l_3 = 235$ mm, $S_1 = 0.1$ mm.