A new type of Fresnel zone plate has been constructed which can focus ultraviolet radiation of any wavelength down to the soft x-ray region. It consists of a set of thin circular gold bands made self-supporting by radial struts, leaving the transparent zones empty. Experimental tests at 6700, 4358, and 2537 A showed that the theoretical minimum angular resolution obeys the Rayleigh criterion, \( \sin \theta_{\text{max}} = 1.22 \lambda/D \). The diameter of the zone plate is \( D = 0.26 \, \text{cm} \) and contains 19 opaque zones, the narrowest of which measured about 20 \( \mu \) across. The zone plate was better than the optimum pinhole in resolution by a factor of about 6 and in speed by a factor of 40. The zone plate produced pictures that compared favorably with those made with a lens of similar focal length and aperture. The lens was about 20 times faster than the zone plate at 4358 A, but at 1000 A the zone plate would have been far faster than the lens. Focusing tests are contemplated at 1000 A and at 100 A where lenses and mirrors, the conventional image-forming devices, may fail. The angular resolution at 2537 A was close to the theoretical value of \( 1.2 \times 10^{-4} \, \text{rad} \) and held over a field of at least \( 1.75 \times 10^{-3} \, \text{rad} \), which is 2.0 times the angle subtended by the sun's disk at the earth. A zone plate telescope, operating in the soft x-ray or extreme ultraviolet region, far above the earth's atmosphere in an orbiting satellite, now seems possible.

INTRODUCTION

This paper describes experiments in the use of a new type of Fresnel zone plate for image formation using visible light and ultraviolet radiation. Tests with visible light and ultraviolet down to 2537 A are described below, followed by a description of our plans to extend these tests down to about 100 A. There is good reason to believe that the same zone plate will operate over a large range of wavelengths from the soft x-ray region through the infrared because its transparent zones are completely open. Therefore, it is opportune to speculate on the possibility of using the zone plate to focus other radiations and particles. First, however, let us consider the motives that prompted us to construct this zone plate for soft x-ray and extreme ultraviolet radiation.

Early research in the means of focusing light was motivated in part by interest in the construction of microscopes and telescopes. These instruments were, after all, the earliest "space probes" into both the intermolecular and extragalactic worlds. Later, the properties of the electromagnetic spectrum outside of the visible region prompted natural extensions of image-forming devices into the near infrared and the ultraviolet.

Today similar motives exist for the construction of microscopes and telescopes to work in the soft x-ray and extreme ultraviolet (hereafter referred to as euv) region. While no definite bounds exist for the euv region we shall limit our study to wavelengths from 10 A to about 1000 A. Although the developments in x-ray microscopy and microradiography have already been responsible for two international congresses,\(^1\) the new interest in telescopes that can operate in the soft x-ray and euv regions stems chiefly from discoveries in astrophysics.

Rocket experiments have already demonstrated that solar, stellar, and interstellar sources of euv exist,\(^2\) but the earth's atmosphere prevents most of the radiations shorter than 3000 A from reaching its surface. The advent of rocket and satellite-borne experiments justifies serious consideration of telescopes and spectrographs sensitive to wavelengths between 1 and 3000 A.

We distinguish between instruments, such as lenses and mirrors, whose main function is to form optical images, and those such as prisms and gratings, whose purpose is to produce dispersion. Our interest for the present is primarily in image formation. The classical means of focusing visible light for image formation involve reflection (mirrors) and refraction (lenses). Proper choice of materials allows the use of reflectors and refractors in the ultraviolet from 3000 A to about 1000 A. Between 1000 and 10 A refraction is out of the question because most materials are opaque to radiation in this region. Special types of glass\(^3\) have been developed which can transmit about 70% at 2200 A through a thickness of 1 mm. But this is exceptional since the transmission of most types of glass drops essentially to zero below 3000 A. Although transmission of wavelengths below 10 A does occur, the index of refraction is so close to unity that refraction is impractical.

The transmission of several materials is shown in Fig. 1.\(^4\) Except for thin films of aluminum, tin, indium, and bismuth, as shown,\(^5\) the materials of which lenses might be made cannot transmit below 1000 A. Below this wavelength, then, lenses are apparently not worth considering.

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\(^{3}\) R. Jastrow, J. Geophys. Research 64, 1647 (1959).


How about mirrors? Anastigmatic image formation with a single mirror requires good reflectivity at normal or near-normal incidence. Recently Hass and his co-workers have developed coatings of evaporated metal films which, when deposited on glass or other materials, produce high reflectivities down to fairly short wavelengths. Figure 2 summarizes the characteristics of some of the best coatings. The data were gathered from many sources and are intended to show general trends rather than authoritative values for which the reader is referred to the original literature, especially for values of the reflectivity of aluminum overcoated with magnesium fluoride at wavelengths below 800 Å. At the present writing it seems safe to say that for wavelengths shorter than 585 Å the Fresnel zone plate may find application as a focusing device, although one must not exclude the possibility of using very large mirrors at normal incidence with very low reflectivities.

For angles of incidence close to 90° it becomes convenient to speak of the "grazing incidence angle," that is, the complement of the usual angle of incidence, which is the angle subtended by the incident ray and the normal to the reflecting surface. For x rays, the phenomenon of total reflection at grazing incidence is well known. The reflectivity is 100% because the index of refraction for most materials is less than unity. Point-to-point grazing-incidence systems for forming images have been built by Kirkpatrick and his students, but reflection from a single curved mirror at grazing incidence is highly astigmatic and can be corrected only by the difficult technique of crossed mirrors. Systems of this type have not received much attention for wavelengths as long as 100 Å but they may have to be considered.

If we restrict ourselves to the less formidable process of image formation by normal incidence reflection at a single surface, we can probably conclude that a region exists, somewhere between 10 and 1000 Å, where image formation by reflection or refraction is either difficult or impossible.

**ZONE PLATE FOR FOCUSING EXTREME ULTRAVIOLET AND SOFT X RADIATION**

Diffraction offers still another way of bending light to produce focusing. Several such methods have been suggested but even the simplest, the Fresnel zone plate, has not received serious use in any region of the spectrum, let alone that between 10 and 1000 Å. Several writers have noted that Fresnel zone plates might be used for image formation in this difficult region, but no successful attempts to build a zone plate for x rays or euv have come to this author's attention.

This paper describes the construction and use of a Fresnel zone plate whose opaque bands are made of thin gold and whose open bands are completely transparent to radiation between 10 and 1000 Å, because they are empty.

A Fresnel zone plate consists of alternately opaque and transparent bands bounded by circles of radii

\[ r_n = r_1 n^1; \quad n = 1, 2, 3, \ldots \] (1)

Its principal focal length \( f \) obeys the simple relation

\[ f \lambda = r_1^2, \] (2)

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**FIG. 1.** The transmissivity of several materials in the ultraviolet. The crystalline materials that might serve for lenses cease to show any appreciable transmission below 1000 Å. The thin metal films could serve as filters.

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where \( \lambda \) is the wavelength, and \( r_1 \) is the radius of the central circle.

In considering the focusing of x rays for microscopy we see that the product \( f \lambda \) can be disturbingly small. For example, in an x-ray microscope one might consider \( f = 1 \) cm and \( \lambda = 1 \) A. This requires \( r_1^2 = f \lambda = 10^{-4} \) cm or \( r_1 = 10^{-4} \) cm or 1 \( \mu \). By Eq. (1) we find that the width of the \( n \)th zone is

\[
s_n = \left[ \left(n + 1\right)^2 - n^2 \right] r_1,
\]

which, for large \( n \) is approximated by

\[
s_n = r_1/2n. \tag{4}
\]

If, for example, \( n = 25 \) and \( r_1 = 10^{-4} \) cm, then \( s_n = 10^{-4}/2(25) \) cm = 10\(^{-8}\) cm or one tenth of a micron. Since the best photographic emulsions (e.g., Eastman 649 spectroscopic plates) have a resolution of about a micron, a zone plate made to these specifications looks impossible indeed. The product \( f \lambda \) needs to be considerably larger before zone plates look feasible. Fortunately, the current interest in the use of x rays and euv in microscopy and astrophysics can lead to larger values of both \( f \) and \( \lambda \).

Consider \( \lambda \) first. Wavelengths much longer than 1 A are important for different reasons. In x-ray microscopy, for example, there has been a trend toward wavelengths approaching 100 A because microscopic objects of biological interest have low atomic numbers and their differential absorption is therefore greater in this region.\(^{19}\) On the other hand, the wavelengths from 10 to 100 A, which are very long x rays, are considered very short compared with the wavelengths of ultraviolet radiation, and are of potential interest in astrophysics. Also, the lines around 150 and 300 A emitted by the sun’s corona could be of interest as sources for telescopic image formation.\(^{20}\)

Now consider the focal length \( f \). In microscopes we want \( f \) to be small, but in a telescope a long focal length is an advantage; in fact, a focal length of 100 cm would be practical and one of 1000 cm, though somewhat large, is not completely beyond possibility if we could build the right kind of platform. The reader must bear in mind that the ultimate platform for telescopes operating in the soft x-ray and extreme ultraviolet regions must be a satellite orbiting above the earth’s atmosphere.

To choose figures of the right order of magnitude, consider \( f = 100 \) cm and \( \lambda = 100 \) A. Using \( n = 25 \) we get \( s_n = r_1/2n = 10^{-8} \) cm or 10 \( \mu \). This value is about 10 times the resolving power of the finest-grained photographic films and makes the construction of a zone plate for soft x rays seem feasible.

However, a zone plate made by exposing a photographic film would be useless in the region between 10 and 100 A because the base on which the emulsion rests and even the emulsion layer itself are practically opaque to these wavelengths.

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Recently we have succeeded in producing a zone plate whose opaque elements are thin concentric bands of gold made self-supporting by the use of thin radial struts (see Fig. 3). The streaks in the photograph are imperfections in the form of very thin filamentlike pieces of gold. The outer diameter of the zone plate is 0.2596±0.0002 cm. The central circle has a diameter of 0.0426±0.0002 cm. The bands are bounded by circles whose radii obey the relation \( r_n = 0.0213\mu^1, \mu = 1, 2 \ldots 38 \).

For a zone plate with a large number of rings the more precise formula for \( r_n \) would be needed,

\[
r_n = (f\lambda)[1 + (n\lambda/4f)]^\mu.
\]

The narrowest band was designed to measure 0.0017 cm in width. Actually it varied between 0.0010 and 0.0020 cm. The zone plate was made for us by the Buckbee Mears Company of St. Paul, Minnesota. It was made by a combination of etching and electrodeposition capable of high resolution. As far as we know this is the first zone plate with transparent regions that are completely open (except for the supporting radial struts). The images formed are very sharp, comparing favorably with those made by lenses of similar focal length and aperture (see Fig. 9). We may suggest one explanation for the good quality obtained with these zone plates as compared with that of the photographically based zone plates. The phase condition that requires an increment of exactly one wavelength between successive transparent bands must be difficult to meet with a film base many microns thick that is not held to optical flatness. This condition is probably less difficult to meet with the new zone plate that has no film base at all.

Since the transparent regions of the zone plate are completely clear they should transmit radiation of all wavelengths. The gold bands are practically opaque to all radiations between 10 and 1000 A; therefore, the zone plate should be able to focus soft x rays and euv in this region. Of course, it can also operate in the visible and infrared regions. It should also work with particles having wavelike characteristics of the proper wavelength, but our present interest is in the soft x-ray and euv region.

Our first zone plate, with which all the tests reported in this paper were made, was designed to have a focal length of about 400 cm at 100 A. At 4000 A its focal length is 10 cm; it thus lent itself very conveniently to tests with visible light. All the tests described below with the exception of those at 2537 A were made with visible light. We plan to continue tests at 100 and 1000 A with newly acquired sources.

### RESOLUTION

Theoretically, the smallest angular separation \( \theta_{\text{min}} \) between two monochromatic point sources at infinity that can just be resolved when imaged by a zone plate, can be shown to obey the Rayleigh criterion for a lens:

\[
sin\theta_{\text{min}} = 1.22(\lambda/D).
\]
that an improvement in resolution has also taken place. The ultraviolet picture is not only the largest, it is the best resolved because it was made at the shortest wavelength.

To exhibit this improvement in resolution apart from the change in magnification, the pictures of Fig. 5 were enlarged photographically so that the distance between squares remained constant. The improved resolution becomes clearer. In Fig. 6 (left), \( p \) equaled 47 cm and \( q \) was adjusted to 7.2 cm, a value that produced the best focus experimentally for a wavelength of 6700 Å. A gelatin filter was used with a pass band of about 400 Å. The object mesh had 4 lines per mm. The angle subtended at the zone plate by two successive centers of open mesh was 0.00053 rad, which is \( 1.68 \theta_{\text{min}} \) for that wavelength.

The center picture of Fig. 6 was made at \( p = 47 \) cm, \( q = 12.5 \) cm, at a wavelength of 4358 Å. A Baird interference filter with a total pass band of about 100 Å was used. Here the angular separation between adjacent centers is \( 2.59 \theta_{\text{min}} \). An improvement in resolution is apparent.

Figure 6 (right) shows an image of the same mesh taken at 2537 Å. The value of \( p \) remained fixed at 47 cm, \( q \) was 27 cm and the angular separation between object centers was now \( 4.44 \theta_{\text{min}} \). The source was a General Electric germicidal lamp number GT4/1. The manufacturer states that this lamp yields 60% of its energy at 2537 Å. A filter was used to remove the visible radiation but the monochromatic effect at 2537 Å is due only to the relative intensity of this line. This demonstrates a point about zone plate focusing that may be useful in ultraviolet and x-ray astronomy: A relatively intense and isolated spectral line may preclude the use of filters. These pictures lead us to believe that the resolution will continue to improve in a predictable way as we go down first to 1000 Å and later to 100 Å with our zone plate.

COMPARISON OF ZONE PLATE WITH PINHOLE AND LENS

Next we compared the image-forming qualities of our zone plate with those of a pinhole. The pinhole size was calculated to give the optimum resolution for the chosen values of \( \lambda, p, \) and \( q \). The optimum diameter for such a pinhole is given by the expression\(^{21}\)

\[
d = 2[0.9pq/(p+q)].
\]  

This may also be written \( d = 2(0.9\lambda/\lambda) \) which, interestingly enough, is very close to the diameter of the innermost circle of a zone plate that gives good focus at the same distances and for the same wavelength. In other words, the pinhole behaves like a zone plate with a single circular opening.

Figure 7 is a picture of a mesh with 4 lines per mm (not visible because unresolved), framed by the lines of a coarser screen with 0.7 lines per mm (poorly resolved), made with the pinhole at a wavelength of 4358 Å. With the same screens as an object and all other factors the same, a picture was taken with the zone plate instead of the pinhole, with the results shown in Fig. 8. The 4 line/mm screen is clearly resolved. The angular separation at \( p = 26.7 \) cm, between adjacent centers of

Fig. 7. An object consisting of a coarse grid, one rectangle of which measures 1.43 mm by 1.72 mm within which there is a fine mesh with four lines per mm, photographed through a pinhole whose aperture was chosen to produce the optimum resolution for the wavelength and distances involved. Only the coarse grid is resolved. Compare this with Fig. 8.

The smaller screen pattern was $5.3 \times 10^{-4}$ rad or $4.5 \theta_{\text{min}}$, as computed for the zone plate. Theoretically, the improvement in resolution of the zone plate over that of the pinhole is a factor $n^3$ of 6.23 for our zone plate of 38 rings. The actual improvement as demonstrated in Figs. 7 and 8 is quite apparent.

The zone plate has another advantage over a pinhole in that the light flux reaching the image is greater with the zone plate, so that much less exposure time is required if all other factors have been left unchanged. In our notation, $n$ stands for the number of circles in the zone plate pattern. Hence $\frac{1}{2}n$ is the number of open zones or bands. The expected improvement in exposure should be $\frac{1}{2}n$ since all the zones have equal areas. For our zone plate, $\frac{1}{2}n = 19$. The actual exposure time used for the pinhole was 40 times greater than that required for the zone plate. Since the densities of the plates were compared only by eye, the experimental values do not seem out of line.

Next we compared the resolution of our zone plate with that of a lens of similar focal length, of aperture equivalent to that of the zone plate. For both the lens and the zone plate $f = 10.2$ cm and $D = 0.26$ cm. Figure 9 (right) shows the image of a screen with 4 lines per mm photographed with the zone plate. Adjacent centers of the object screen subtended an angle equal to 2.6 $\theta_{\text{min}}$. Other constants were $p = 47$ cm, $q = 13$ cm, and $\lambda = 4358$ Å. The lens had a simple plano-convex form. If a highly corrected camera lens had been chosen the comparison might have favored the lens, but since it was stopped down to $f/18$ and the field covered was not very large, it was not subjected to undue demands as an image-forming device. We can conclude that the zone plate image compares favorably with that of the lens.

The zone plate was about 20 times slower than the lens for the following reason: In the vector diagram representing the amplitude of the waves that pass through successive Fresnel zones, the continually changing phase produces a spiral (Fig. 10). In Fresnel's treatment of the contributions of successive circular zones, the amplitude of the contribution due to one zone is represented by the vector $AB$. The length of the arc $ABC$, starting at $A$ and ending at $C$, would be proportional to the amplitude produced by two zones of a lens (since the lens has the property of orienting all the infinitesimal vectors in the same direction). The ratio of these amplitudes is approximately $\pi$. The ratio of intensities would be $\pi^2$. This argument would hold for a perfect zone plate, that is, one in which the width of an opaque band never exceeded the mathematically computed value of $s_n$.

With the real zone plate, the widths of all the gold bands are greater than the theoretical value of $s_n$, thereby reducing the intensity of the light gathered at the focus. A perfect zone plate introduces the factor $\pi^2$ and inaccuracies in manufacture introduce a factor of about 2, making the ratio approximately $2\pi^2$, which is about 20. This means that a single zone plate is not a very efficient gatherer of light when compared with a lens. However, with sufficiently intense sources it can produce images with good resolution. Certainly the resolution and light-gathering power are much better than those of a pinhole. We conclude that the zone plate is much better than a pinhole both in resolution and light-gathering power; it is comparable to a lens in resolution but about 20 times slower than a lens of similar aperture when used with visible light.

To overcome the intensity limitation the method of superposition of images could be used. It has been shown that $n$ pictures of the same object each exposed...
for a time \( t \) (where \( t \) would yield an underexposed negative) give negatives which when superimposed produce an image comparable in exposure to that of a single picture exposed for a time \( nt \). Instead of photographic recording some form of electronic readout might be used to integrate the signals from \( n \) zone plates and produce an exposure in \( t \) seconds that ordinarily would have required \( nt \) seconds. Because noise increases along with the signal, the advantages in resolution are not simply multiplied \( n \) times. Nevertheless, a gain does result from this technique and since zone plates may eventually be produced fairly cheaply, it is conceivable that the signals received simultaneously from many zone plates could be integrated advantageously.

CONCLUSION

In summary, then, we have produced a Fresnel zone plate consisting of a set of gold bands supported by radial struts, leaving alternate bands almost completely empty, thereby making possible the transmission and focusing of soft x rays and extreme ultraviolet radiations. We have tested it in visible light and in the ultraviolet only as far as 2537 Å. All tests indicate that the experimental resolution and transmissivity are what one would expect from simple theoretical considerations. This means that from the point of view of resolution and exposure the zone plate is a great improvement over a pinhole. It compares favorably in resolution, but not in speed, with a simple lens, if we make the comparison in the visible region. Below 1000 Å, however, the speed of the zone plate would be thousands or millions of times greater than that of a lens made of any known material. All the experimental evidence now in existence seems to indicate that there is a region in the euv where the zone plate could serve as the only practical focusing device, because the reflectivity and the transmissivity of known materials would make the use of lenses or mirrors at normal incidence either difficult or impossible.

The lines of future research now seem clear. First, experiments to focus soft x rays and euv radiation between 10 and 1000 Å should be made since in this region the conventional methods of image formation seem likely to break down. Next, as the zone plate is a highly chromatic device, sources of essentially monochromatic radiation must be found for it. Several schemes suggest themselves at once. For example, limiting stops placed strategically can absorb the radiation of one wavelength while almost all else proceeds essentially unaffected. If we could precede the circular zone plate by similarly self-supporting diffraction gratings, new types of euv spectrographs with little or no astigmatism might become feasible.

The uses to which these zone plates may be put in the astronomical telescopes of the future will be determined by the problems that need to be explored in astrophysics. Because of the great intensity available from the sun some early solar experiments would probably be in order. The isolated lines at around 150 and 300 Å in the spectrum of the sun's corona seem ideal for an

\[ \text{FIG. 9. Comparison of a lens and a zone plate as image forming devices near the limit of resolution. On the left is a picture of a fine mesh screen taken with a zone plate. On the right is a picture of the same screen taken with a plano-convex lens of similar aperture and focal length. All other factors were kept the same except exposure which was 20 times greater for the zone plate. The wavelength used was 4358 Å. If the exposure comparison had been made at 1000 Å the results would have been much more favorable for the zone plate.} \]

\[ \text{FIG. 10. The vector } AB \text{ represents the resultant of the infinitesimal vectors contributed by the subzones of one open Fresnel zone in a zone plate. The length of the arc } ABC \text{ represents the amplitude of the contributions due to two successive Fresnel zones in a lens. The continuously changing thickness of a lens causes the infinitesimal vectors to line up since they all have the same phase. The ratio of the length of the arc } ABC \text{ to the length } AB \text{ is approximately } \pi. \]

\[ \text{[\text{Giacconi and B. Rossi, J. Geophys. Research 65, 773 (1960).}]} \]
experiment with a zone plate telescope "tuned" to one of these lines. Next in intensity—and in possible interest—might be the moon and the planets. The present limitations on speed indicate that we must develop means of greatly increasing the effective speed of a zone plate device before it can be useful in gathering euv and x rays from the stars.2

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INTRODUCTION
A THEORETICAL investigation1,2 of diffraction images of nonself-radiant, disk-shaped particles viewed under full Köhler illumination has provided a function which can be used to describe the diffraction image formed by an Airy-type objective when the object is an extended, uniformly self-radiant circular disk. This function determines the time-averaged irradiance3 in the diffraction image versus radial distance from the center of the image. Within the approximations of the theory, details of the diffraction images are found to depend only upon the ratio of the size of the geometrical image to the size of the Airy unit $r_a$ in the image space. This Airy unit is defined by the equation $2\pi r_a \rho_m \sin \theta_m = 3.8317 \lambda$, where 3.8317 is approximately the first nonzero root of $J_1(x)$; $\rho_m$ is $\sin \theta_m$; $\theta_m$ is the maximum angular opening of the objective with respect to its image space; $n$ is the refractive index of the image space (taken as unity here); and $\lambda$ is wavelength.

A special solution for computing in a more convenient manner the irradiance in the image at points corresponding to the geometrical edge of any disk has been added.

This work complements the investigation by H. Nagaoka,4 who obtained curves of irradiance in telescopic images when the radius of the source is many times the limit of resolution.

3 Formerly called energy density.
4 H. Nagaoka, Phil. Mag. Ser. 5, 45, 1 (1898).

EXPRESSION FOR IRRADIANCE
Nagaoka’s expression for the distribution of irradiance $H(L)$ in the plane of the sharply focused image of a self-radiant disk can be written in the form

$$M^2 H(L)/\rho_m^2 = \int_0^\infty \int_0^{2\pi} [J_1(2\beta\omega)/\omega^2] \cos \phi \, d\phi \, d\omega, \quad (1)$$

in which

$$\omega = (L^2 + \rho_m^2 - 2Lt \cos \phi)^{1/2}, \quad \beta = 3.8317, \quad (2)$$

where $L$ is distance in Airy units from the center of the diffraction image, $K$ is the radius (in Airy units) of the geometrical image of the self-radiant disk and $M$ is the magnification ratio of the objective.

Equation (1) can be derived from modern theories of image formation by requiring that the self-radiant disk shall radiate uniformly in a Lambertian manner and that the objective shall be of the Airy type, i.e., shall image a small dipole radiator as a distribution of amplitude and phase with the form $J_1(ax)/(ax)$.

Integration of Eq. (1) can be carried out as shown by Osterberg and Smith,6 for the double integral in the right-hand member is just the function $I_{ud}$ which arises in the theory of images of nonself-radiant disk-shaped particles viewed with full Köhler illumination. The behavior of $M^2 H(L)/\rho_m^2 = I_{ud}$ for radii up to 4 Airy units is shown in Fig. 1, which is a plot of values

6 $K$ is also the radius of the self-radiant disk in Airy units with respect to the object space. However, the Airy unit with respect to the object space is $1/M$ times the Airy unit $r_a$ with respect to the image space.