E. THEORY AND TECHNIQUE OF REFLECTION X-RAY MICROSCOPY

THE OBLIQUITY ABERRATION OF REFLECTION X-RAY MICROSCOPY*

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ABSTRACT

The obliquity aberration causes the image plane to recline with respect to the principal ray. The insertion of a stop of vanishing aperture at the proposed distance \( D_o = \frac{R}{\sin i} \) in image space is shown to result in the erection of one image point into the Gaussian plane. For systems of finite magnification \( M \) and mirrors of finite length \( s \) the position of the stop is shown to be \( D = D(M, f, s) \) where \( f \) is the focal length. The latter function reduces to \( D_o \) when \( M \) is infinite and \( s \) is zero. The analysis is further extended to include the positioning of a finite aperture stop. Extensive ray tracing with the aid of an IBM 650 computer has resulted in a more complete understanding of the role of aperture stops and the prediction of the performance of a complete reflection X-ray microscope.

INTRODUCTION

The focusing X-ray microscope described by KIRKPATRICK and BAER \(^1\) consists of two concave X-ray reflectors mounted one behind the other. The reflectors which may be sections of much larger polished glass mirrors are oriented so that the tangent planes at the center of each mirror are mutually perpendicular. This disposition succeeds in removing the extreme astigmatism which is characteristic of a single reflecting mirror. Because of the extreme astigmatic nature of a single reflector the optical analysis may be confined to two dimensions in the meridian plane, where the image and object positions are related for a mirror of small length by

\[
\frac{1}{p} + \frac{1}{q} = \frac{2}{R \sin i} = \frac{1}{f_m}
\]

with \( R \), the radius of the mirror segment, \( f_m \), the focal length in the meridian plane and \( i \), the angle of grazing incidence (complement of the ordinary angle of incidence). For total reflection of X-rays to take place, the angle of grazing incidence \( i \) has to be less than the critical angle \( i_c \), which is at most a few degrees. The latter is a function of the wavelength and the electron density of the reflecting surface. For glass and radiation of wavelength 1.54 Å, the critical angle is about 4.5 milliradians. With radiation of 8.34 Å incident on gold, the critical angle increases to approximately 35 milliradians.

Beside the meridional focal length \( f_m \) there exists a sagittal focal length \( f_s \), which is many thousands of times the meridian focal length for a typical X-ray microscope system. The sagittal focal length is given by

\[
f_s = \frac{R}{2 \sin i}
\]

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GEOMETRICAL ABERRATIONS

In common with other optical devices, the X-ray microscope does not produce images which are exact reproductions of the object. It further differs from most other optical systems by not possessing a symmetry axis. Accordingly it is to be expected that the reflection X-ray microscope will produce the common geometrical aberrations, such as spherical, comal, etc., together with some aberrations not possible in symmetrical systems.

The formation of spherical aberration by a concave spherical reflector is illustrated in Fig. 1. A narrow pencil of rays from the point object O, incident on a small section of mirror at C is imaged perfectly at I according to Gaussian optics.

![Diagram](image)

**Fig. 1.** Spherical aberration formed by a concave spherical reflector.

However, it is seen that another narrow pencil from O, incident on the reflector at M, will be focused at I'. A photographic plate placed in coincidence with the Gaussian plane would thus record an image of width δ. The length δ is a measure of the transverse spherical aberration. The defect could also be specified by \( \delta = \Pi' \), the *longitudinal* spherical aberration. The transverse and longitudinal spherical aberrations are related by the aperture angle \( \alpha' \),

\[
\delta = \alpha' \Delta.
\]

KirKPATRICK and BAEZ show that, for large magnification \( M = q/p \) and a small mirror segment \( CM = s \), the transverse spherical aberration is

\[
\delta_{ts} = \frac{3}{2} \frac{Ms^2}{R}.
\]

The projection of the mirror length s into the aperture plane of Fig. 1, located adjacent to the mirror, shows that for large magnification M the separation of the pencils in the aperture plane is given approximately by \( r = si \) where \( i \) is the angle of incidence of the principal reference ray OCI. The transverse spherical aberration

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According to Dymond \( \tan \gamma = \frac{i(R - 2D)}{iR + D} \) for \( M \to \infty \).

\[ \Rightarrow i \quad \text{for } D = 0 \]

but Dymond's \( \tan \gamma \) and so on is all in object space in image space \( \tan \delta \to \frac{i}{M} \) by Helmholtz Lagrange theorem.

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Fig. 2. Obliquity aberration.

aberrations in the origin of the Gaussian (dotted) plane. Because of the obliquity aberration, the bundle of rays from the other end of the object is focused behind the Gaussian image plane. It is clear from Fig. 2 that one bundle of rays will intersect the Gaussian plane over a distance \( \delta \) while the other will have an interception of zero extent. Ray bundles from intermediate field points will correspondingly intercept the Gaussian plane with overlapping intercepts varying in extent from zero to \( \delta \). Because of obliquity, the transverse aberration \( \delta \) associated with any one field point is a function of its perpendicular distance \( h \) measured from the principle reference ray in object space and the aperture \( r \) of the system.

In Fig. 2 the longitudinal aberration due to obliquity alone is approximately given by \( \Delta = \frac{Mh \gamma}{\delta} \). As stated above \( \gamma \approx i/M \) so that \( \Delta \approx M^2 \gamma /h \). The aperture angle \( \alpha' \approx r/\delta \) and \( q \approx iRM/2 \) so that \( \alpha' \approx 2r/iRM \). The transverse aberration due to obliquity alone is approximately

\[ \delta_{1n} = \Delta \cdot \alpha' = (M^2 \gamma /h)(2r/iRM) = (2M^2 \gamma /h)R = a_{1n} hr. \]

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The transverse aberrations $\delta_{2}$ and $\delta_{11}$ may be considered as terms in a power series expansion of the total transverse aberration $\delta_{t}$ which is expressed as

$$
\delta_{t} = a_{00} + a_{01}r + a_{02}r^{2} + \cdots
$$
$$
\cdots + a_{10}h + a_{11}hr + a_{12}hr^{2} + \cdots
$$
$$
\cdots + a_{20}h^{2} + a_{21}h^{2}r + \cdots
$$
$$
\cdots + a_{22}h^{2} + \cdots.
$$

Since the optical system is unsymmetrical, the power series may contain even as well as odd powers.

The definition of the Gaussian plane requires that $\delta$ be zero when $h$ and $r$ are respectively zero. Thus the coefficient $a_{00}$ should be zero. It is possible to interpret other terms of the power series as due to a particular type of geometrical aberration. When $\delta$ depends only on $r^{2}$, the coefficient $a_{02}$ would be called the primary spherical aberration coefficient. The coefficient $a_{11}$ is associated with the obliquity defect while the coefficient $a_{12}$ is associated with coma and $a_{21}$ with curvature of field.

**Correction of obliquity**

The usual approach to correcting alterations in an optical system is either an alteration of the optical surfaces, the insertion of apertures, or both. Dyson considered the insertion of an aperture of vanishing width at a distance $D = Ri/3$ from the mirror in image space. It will be shown that a narrow aperture at this position will erect but one conjugate field point and then only for a mirror of vanishing length, adjusted for infinite magnification.

To determine the position $D$ of a narrow aperture which will correct the obliquity defect, consider the object $OA$ and the erect image $IB$ which are located at distances $p_{c}$ and $q_{e}$ respectively from the mirror center $C$ of Fig. 3. The object $OA$ has been erected perpendicular to the principal reference ray. It is desired to find the condition...
or position D of the aperture which will result in the image being perpendicular to the principal reference ray in image space. The extreme rays drawn from the line object of Fig. 3 are considered to be narrow bundles of rays which after reflection at C and M respectively are brought to a focus in the extreme points of the image BI after being forced to cross over at the narrow aperture opening E. If the ray \( p_1 \) is incident at an angle \( i_1 \) on the concave reflector of radius \( R \) while the ray \( p_f \) is incident at an angle \( i_f \) at point M it follows from the geometry of Fig. 3 that for small angles \( i_f = \beta + \theta \) with \( \theta \approx \Theta/2 \) and
\[
\theta \approx s/R
\]
or
\[
i_f = \beta + s/2R.
\]
In triangle MCE the following relationship holds
\[
\beta + q - \frac{\theta}{2} - i_e = 0.
\]
The application of the law of sines to triangles MCE yields
\[
\frac{s}{q} = \frac{D}{\beta}
\]
In order that the object point O be imaged in the Gaussian plane through BI the following conditions must hold at small angles.
\[
\begin{align*}
    p_e &\approx p_f + s \\
    q_e &\approx q_f - s \\
    i_f + i_f &\approx 1/i_e \\
    i_f + i_e &\approx 1/i_e
\end{align*}
\]
where \( f_e = R i_f/2 \) and \( f_e = R i_e/2 \). In the subsequent analysis it will be convenient to have equation (12) in the two forms
\[
\begin{align*}
    p_e &\approx \frac{1 + M}{M} \cdot i_e \\
    q_e &\approx (1 + M) \cdot i_e
\end{align*}
\]
where
\[
M = q_e/p_e.
\]
Also equation (13) is better expressed as
\[
\begin{align*}
    p_f &= \frac{q_f}{q_f - i_f} \approx \frac{(q_e + s)f_e}{q_e + s - i_f}.
\end{align*}
\]
Successive substitutions in eqns. (10) of \( p_e, p_f, q_e, i_e \) and \( i_f \) from equations (14), (16), (15), (13) and (7) respectively and of \( f_e \) and \( f_e \) results in the following expression for the angle \( \beta \)
\[
\beta = \frac{4(1 + M)^2 i_f^2 + (3 - 5M)(1 + M)i_f^2 - 4M^2}{2sfR(1 + M)^2}.
\]
Another expression for \( \beta \) can be developed from eqn. (8) by first eliminating \( q \) with eqn. (9) and subsequently using \( \theta \approx s/R \) and \( f_e = R i_e/2 \) as substitutes for \( \theta \) and \( i_e \) with the result that
\[
\beta = \frac{D(s + 4f_e)}{2R(s + D)}.
\]
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The final step is the equating of eqns. (17) and (18) and solving for \( D \) to give

\[
D = \frac{4(t + M) f^2 + (3 - 5M)(t + M)f^2 - 4Ms^2}{2(t + M)(3M - 1)f + 4Ms}
\]

For infinite magnification eqn. \( (19) \) reduces to

\[
D = \frac{R_{1e}}{3} - \frac{5}{6} s.
\]

If \( s \) approaches zero, then \( D = R_{1e}/3 \), which is just Dyson's result for a narrow aperture. For unity magnification and zero length of mirror

\[
D = \frac{1}{6} + \frac{1}{6} l_e = R_{1e},
\]

and the aperture would coincide with the image position.

With due consideration of Fig. 3 and eqn. \( (19) \) it becomes evident that the infinitesimal ray bundles originating at intermediate object points must intersect the mirror at different values of \( s \) and cross the principle reference ray at different points if they are to focus in the Gaussian plane \( \Pi \). Thus a single narrow aperture will cause but one field point to be focused in the Gaussian plane. It would seem that an infinite number of narrow apertures would be required if each and every point of the field is to be focused into the Gaussian plane and a completely erect image obtained. The prospect of accomplishing this looks discouraging until a few key rays are drawn as shown in Fig. 4. The intersections \( a \) and \( b \) are readily determined for each extreme ray through an application of eqn. \( (19) \).

![Fig. 4. Effect of several apertures on focusing.](image)

In Fig. 5, \( D \) is plotted versus \( s \) for various magnifications \( M \) with \( f_e = R_{1e}/2 \) constant = 17.875 which corresponds to \( R = 325 \) cm and \( l_e = 0.55 \) radians.

The distance \( D \) for the intersections \( a \) and \( b \) can conveniently be read from Fig. 5 for a reflector of maximum length \( 2s \). It is obvious in Fig. 4 that the placing of narrow apertures at the intersections \( a \) and \( b \) will not be satisfactory because of the

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blockage of rays. Perhaps half-apertures (single knife-edges) at the three intersections a, b, and c would still insure that the rays cross at the proper distance D? The latter solution is tenable until one plots the paths of ray from intermediate field points. It is found that they will intersect the principal reference ray at points between a and b with the end result that the knife edges previously placed at a

and b would cut off rays from intermediate points of the field! Thus if one wishes to image intermediate field points, the half-apertures originally placed at a and b of Fig. 4 have to be moved toward one another until they finally coalesce into one at some intermediate point almost directly above point c. It now becomes clear that the half aperture between a and b together with the half aperture at c constitute but one finite aperture as distinguished from the narrow (infinitesimal) apertures previously considered but tacitly assumed to be finite in practice! However, this finite aperture is not of itself capable of directing the limiting rays of Fig. 4 to cross the principal reference ray at the correct point as given by eqn. (19). To bypass some of the original logic, let it be assumed that a half-aperture is placed at point d. The edges of the half-apertures c and d determine a straight line which coincides with the desired direction of the extreme ray AA'. Likewise the single edge at c and the edge d determine the direction of the remaining extreme ray BB' after an intermediate reflection. Thus a finite aperture (ab), c and a half-aperture d serve the function of the infinite number of narrow apertures placed along the principal reference ray as required by eqn. (19). It appears that the lower edge c of the finite aperture and the edge d of the half-aperture define the correct direction for the extreme rays; in one case, after one intermediate reflection.

If each image point is considered the source of a bundle of rays of angular width α' which are traced backward through the system, it is evident that one end of the image will be “vignetted” by the combination of the finite aperture and the half-aperture. Before becoming alarmed at this and other possible ill effects it would be wise to determine more completely by ray tracing exactly what happens in detail.

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(a) Image contours

A preliminary step in the determination of how well the expression \( D = D(M, s, f_s) \) removes obliquity. Again the reverse approach is helpful. If obliquity is to be totally absent, then the infinitesimal bundle of rays from any image point in the Gaussian plane which strikes the reflector at a point \( s \) should cross the principle reference ray at a point \( D \) cm from \( C \) as given by eqn. (19). The computation starts with a directed ray passing through the desired Gaussian image point and striking the reflector at \( s \) and crossing at \( D \) as computed from eqn. (19). This is sufficient to fix the reverse reflected ray direction so that its intersection with the Gaussian plane in object space can be computed. From the latter intersection point and the reflector intersection point at \( s \) the object distance \( p_t \) can readily be computed. An application of the focusing condition expressed by eqn. (1) allows the determination of \( q_s \) which should fall in the Gaussian image plane. The above calculation is repeated for values of \( s \) between \(-0.8 \) cm to \(+0.8 \) cm. It is an easy matter to compute the coordinates of each image and object point in a \( \xi, \eta \) co-ordinate system whose origin is at \( C \), the \( \xi \) axis tangent to the reflector at \( C \) and the \( \eta \) axis along the radius \( R \). The image contours of Fig. 6 were computed for three different cases. Case I was for \( R = 850 \) cm and \( i_s = 0.015 \) radians; Case II for \( R = 325 \) cm and \( i_s = 0.015 \) radians; Case III for \( R = 325 \) cm and \( i_s = 0.055 \) radians. A Gaussian image plane if drawn in Fig. 6

![Image](image_url)

Fig. 6. Two-dimensional image contours of an erect line object.

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would be almost perpendicular to the $\xi$-axis and touch each image contour at its approximate center.

The width of field corresponding to the image contour of Case I is $192 \mu$. It should be noted that the width of mirror (28) used for this case was only $1.0 \text{ cm}$. The width of field corresponding to the image contour of Case III is $1288 \mu$. It is now evident that an image relatively free of obliquity may be produced by causing the rays from a given field-point to cross the principal reference ray of Fig. 4 at the point determined by eqn. (19). The resulting image planes do not depart significantly from their Gaussian planes. It is seen that some deviations are less than the thickness of the photographic emulsion which would ordinarily be positioned in or close to the Gaussian plane to record the image. It should be remembered that each image point has been computed with zero spherical aberration because an infinitely narrow beam or ray was used in the calculation.

(b) Total geometrical aberration

A separate ray-tracing study using the IBM 610 computer was next undertaken to determine the total geometrical aberration produced by the system. Each object point was considered the source of a divergent bundle of rays with uniform density and of sufficient divergence to fill the aperture. In the actual machine calculation the program was arranged so that a larger than necessary bundle of equally spaced rays could be sent through the computer. Mathematical limits representing the edges of the full and half apertures would automatically reject those rays which should hit the sides of the physical apertures. The computer would then start over with the next adjacent ray.

Assuming object and image planes have been erected, then the path of any ray through the optical system is completely determined if $h$ the distance of the object point from the principal reference ray and $r$ the coordinate of the rays' intersection with an aperture plane are specified. In Gaussian optics, a field-point at $h$ would intersect the Gaussian image plane at a distance $Mh$ units from the principal reference ray. An aberrant ray intersects the Gaussian plane with an error $\delta$, which may be expressed as a function of $h$ and $r$ by the power series of eqn. (6). Its coefficients are of course functions of various system parameters such as object distance, magnification, angle of incidence $i$, radius $R$ of the reflector, etc. Provided the foregoing analysis of obliquity is correct, the coefficient $a_4/4$ should be extremely small.

An attempt to determine the coefficients of eqn. (6) was made assuming that terms beyond the third order could be neglected. The computation was carried out for a mirror of radius $R = 325 \text{ cm}$ and $i = 0.055 \text{ radians}$. Sufficient input information covering a large range of field $h$ and aperture $r$ was available from the previous ray tracing study. A standard method was used for triangulation of the matrix in $(h, r)$ and the coefficients were improved in accuracy by a technique due to CROUT. A considerable loss in precision may result through cumulative round-off errors and failure to evaluate the coefficients in the proper order. The resulting coefficients are tabulated in Table I. They should be divided by the magnification.

(c) Intensity distributions

The final study is concerned with the distribution in the image contour of a uniform angular distribution of rays emanating from one field-point. Once tha

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Table I

\[
\begin{array}{ccc}
\alpha_1 &=& -0.0086297 \\
\alpha_2 &=& 0.0010645 \\
\alpha_3 &=& 3.9670 \\
\alpha_4 &=& 1.8874 \\
\alpha_5 &=& -0.53659 \\
\alpha_6 &=& 22.387 \\
\alpha_7 &=& -680.60 \\
\alpha_8 &=& 36.179 \\
\alpha_9 &=& 4.4603 \\
\end{array}
\]

Coefficients of eqn. (6) have been determined, it is an easy step to program the computer for calculating a large number of intersections for a particular case. The linear density of the intersections was converted to an arbitrary intensity scale and plotted as a function of distance from the intersection of the chief ray (zero aperture) as shown in Figs. 7 and 8. In the case of Fig. 7 the above process was repeated for another field-point displaced 0.1 \( \mu \) from the original point at \( h = 400 \mu \). While time did not permit making computations for two points closer than 0.1 \( \mu \), it is clear that

![Fig. 7. Intensity vs. image field in microns. Object points 0.1 \( \mu \) apart.](image)

![Fig. 8. Intensity vs. image field in microns. Object points 0.05 \( \mu \) apart.](image)

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adopting almost any reasonable resolution criterion, the geometrical resolution for this case is much less than 0.1 μ. In the computations leading to Fig. 7 the original point was taken at the center of field h = 0 and the displaced point at a distance of 0.05 μ from the center of field.

REFERENCES

\[\theta = \frac{s}{r}\]

\[\Delta = \frac{1}{2} \theta\]

**Eqn 7**
\[\theta + \phi = \beta + \frac{1}{2} \theta = \beta + \frac{1}{2} \frac{s}{r}\]

**\(\Delta MCE\)**
\[180 - (\beta + \phi) = (i_c - \frac{\theta}{2}) + 180 - 2i_c\]

**Eqn 8**
\[\beta + \phi - \frac{\theta}{2} - i_c = 0\]

**Eqn 9**
\[\Delta MCE \text{ Sine rule } \frac{S}{\phi} = \frac{D}{\beta}\]

10 & 11 obvious
\[\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \frac{1}{p} = \frac{1}{f} \quad \frac{1}{p} = \frac{1}{f}\]

**Eqn's 14, 15**
\[\therefore p = f\left(\frac{M+1}{M}\right) \quad \therefore q = \frac{1}{f}\]

\[\frac{1}{p} = \frac{q - f}{q} \quad \therefore \frac{1}{p} = \frac{q - f}{q} \quad \therefore \frac{1}{p} = \frac{q - f}{q}\]

\[\therefore pf = \frac{q - f}{qf - f} = \frac{q - f}{qf - f} \quad \therefore pf = \frac{q - f}{qf - f}\]
\[ P_c = P_f + S \]

Substituting from 14
\[ P_f \text{ from 14} \]
\[ P_f = 16 \]

\[ \frac{1+M}{M} f_c = \frac{(q_c + S) f_f}{q_c + S - f_f} + S \]

\[ q_c = (1+M) f_c \]
\[ \frac{1+M}{M} f_c = \frac{[(1+M) f_c + S] f_f}{[(1+M) f_c + S - f_f]} + S \]

\[ f_f = \frac{R f_f}{2} \text{ and } f_f = \beta + \frac{1}{2} \frac{S}{R} \]

\[ \frac{1+M}{M} f_c = \frac{[(1+M) f_c + S] R}{2} \left( \beta + \frac{1}{2} \frac{S}{R} \right) \]

\[ \frac{1+M}{M} f_c + S - \frac{R}{2} \left( \beta + \frac{1}{2} \frac{S}{R} \right) + S \]

Let \((1+M) f_c = k\)

\[ \left( \frac{k}{M} - s \right) \left( k + s - \frac{R}{2} - \frac{s}{4} \right) = \left( k + s \right) \left( \frac{R}{2} + \frac{s}{4} \right) \]

\[ \frac{k^2}{M} + \frac{k s}{M} - \frac{R k}{2} - \frac{s k}{4} - \frac{s k}{4} - \frac{s k}{4} - \frac{s k}{4} - \frac{s k}{4} - \frac{s k}{4} = \frac{R k}{2} \left( \frac{1}{M} + 1 \right) \]

\[ \beta = \frac{\left( \frac{k}{M} - s \right) (k+s) - \frac{s k}{4}}{M} \left( \frac{1}{M} + 1 \right) \]

\[ \frac{R k}{2} \left( \frac{1}{M} + 1 \right) \]

\[ \frac{k^2}{M} + \frac{k s}{M} - \frac{s k}{4} - \frac{s k}{4} - \frac{s k}{4} - \frac{s k}{4} = \frac{R k}{2} \left( \frac{1+M}{M} \right) \]
\[ k = (1 + M) f_c \]

\[ \beta = \frac{4k^2 + 4fs - 4Mks - 4Ms^2}{2Rk(1 + M)} \]

\[ \beta = \frac{4k^2 + 3fs - 5Mks - 4Ms^2}{2Rk(1 + M)} \]

Eqn. 17: \[ \beta = \frac{4(1 + M)^2 f_c^2 + (3 - 5M)(1 + M) f_c s - 4Ms^4}{2Rf_c (1 + M)^2} \]

\[ \beta = \frac{\beta}{2} + \frac{i_c - \phi}{2} \]

\[ \beta = \frac{\theta}{2} + \frac{i_c}{2} - \frac{5\beta}{D} \quad \text{using} \ 9 \]

\[ \beta = \frac{\theta}{2} + \frac{i_c}{2} - \frac{5\beta}{D} \quad \text{using} \ 9 \]

\[ \beta = \frac{2}{2R} + \frac{2f_c}{R} \quad \text{using} \ 9 \]

\[ \beta = \frac{D(s + 4f_c)}{2R(s + D)} \]

Re-coupling: \[ 2R(s + D) \beta = D(s + 4f_c) \]

\[ 2R \beta + 2RD\beta = D(s + 4f_c) \]

\[ D(2R \beta - s - 4f_c) = -2R^2 s \beta \]

Let: \[ 2R \beta = \frac{A}{B} \]

\[ D = \frac{A}{2R^2} \frac{\beta}{B} \quad \text{from} \quad A - s - 4f_c \]

\[ = -\frac{A \beta}{B} \left( \frac{A - B - B - 4f_c}{B} \right) \]

\[ = -\frac{A \beta}{B} \left( \frac{A - B - B - 4f_c}{B} \right) \]
\[ D = \frac{4 (1+M)^2 f_c^2 + (3-5M)(1+M) f_c s - 4Ms^2}{2 (1+M)(3M-1) f_c + 4Ms} \]

For \( M = \infty \) only the \( M^2 \) terms count

\[ D = \frac{4M^2 f_c^2 - 5M^2 f_c s}{6M^2 f_c} \]

\[ D = -\frac{2 f_c}{3} - \frac{5}{6} s = \frac{R_i}{3} - \frac{5}{6} s \quad \text{eqn. 20} \]

as \( s \rightarrow 0 \) \( D \rightarrow \frac{R_i}{3} \) Dyson's result, with NARROW aperture.